Propagation of Tsunami-like Surface Long Waves in the Bays of a Variable Depth

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Within the framework of the nonlinear long wave theory the regularities of solitary long wave propagation in the semi-closed bays of model and real geometry are numerically studied. In the present article the zones of wave amplification in the bay are found. The first one is located near the wave running-up on the beach (in front of the bay entrance) and the other one – in the middle part of the sea basin. Wave propagation in these zones is accompanied both by significant rise and considerable fall of the sea level. Narrowing of the bay entrance and increase of the entering wave length result in decrease of the sea level maximum rises and falls. The Feodosiya Gulf in the Black Sea is considered as a real basin. In general the dynamics of the waves in the gulf is similar to wave dynamics in the model bay. Four zones of the strongest wave amplification in the Feodosiya Gulf are revealed in the article. The sea level maximum rises and extreme falls which tend to grow with decrease of the entering wave length are observed in these zones. The distance traveled by the wave before the collapse (due to non-linear effects), was found to reduce with decreasing wavelength of the entrance to the bay (gulf).

Keywords: nonlinear long waves, quadratic bottom friction, wave propagation in the bays, bays of model geometry, the Feodosiya Gulf, numerical solutions.

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Introduction. Basin depth change, gulfs, straits and other irregularities of bathymetry and basin boundaries have an impact on the propagation of long waves in the coastal areas of sea basins [1, 2]. Analysis of the wave propagation and amplification features turns to be important not only for the study of common physical laws of the tsunami-like wave evolution, but also to improve forecasting methods and tsunami zoning of the World ocean coast.

One-dimensional propagation of long waves in channels, straits and bays, taking into account changes in the geometric parameters of water bodies have been studied in a great variety of papers (e. g. [1, 2]). In the articles [3 – 6] within one-dimensional model framework the numerical and analytical evaluation of the amplitude characteristics of waves in the channels and straits are presented. The dependence of the wave maximum heights and horizontal velocities on the parabolic cross-section channel depth was studied in the articles [7, 8]. In one-dimensional problems the averaged cross-section channel displacement of the fluid free surface and horizontal velocity are applied. Such problems are easy to implement and allow describing the propagation of long waves in channels and bays. However, they do not take into account the bottom topography in cross-section. It can significantly affect the process of wave propagation in the shallow channels, bays and gulfs. In the bays of cross-section shape, being different from the rectangular one, the new features of the wave evolution appear. In such cases it is difficult to find an analytical solution that takes into account the basin cross section effects. It requires application of the numerical methods.

The two-dimensional tsunami wave propagation in the channel [9] and through the strait of rectangular cross-section [10] was numerically studied. However, cross-section bathymetry of the real bays and straits has a more complex shape.

Many articles have been devoted to the numerical description of the propagation of long waves in the real basins [11, 12].

Studying the propagation of tsunami waves in various bathymetry basins, the diversity of shapes of lateral boundaries of the oceans and seas should be taken into account. Considerable wave amplification can occur in bays and gulfs with a specific shape. In this case the nature of wave propagation is difficult to predict. Seiche oscillations [13, 14] and various trapped waves [15] can appear in enclosed and semi-enclosed bodies of water. Gulfs often have a shape close to a semi-circular. In the Black Sea these are particularly the Feodosiya Gulf and Gelendzhik Bay.

Below the study of nonlinear long wave propagation in the semi-enclosed bays of the model geometry is carried out. In addition, the Feodosiya Gulf is considered as the real water area. The most dangerous zones of the wave amplification in the Gulf are discovered. Also, the dependencies of the sea level maximum rises and falls on the entering tsunami-like wave length are calculated.

Mathematical formulation of the problem. In the horizontal plane *Oxy* the bay of model geometry, limited by semi-circular shape coastline is considered (Fig. 1, *a*). The bay depth is maximal at the entrance and is 36 m away from the coast. The minimum depth along the bay coast was taken to be 4 m. The solitary wave propagation in a bay is studied. Nonlinear dynamics of long waves in two dimensions taking into account the quadratic bottom friction is described by a system of three equations

$$u_t + uu_x + vu_y + g\zeta_x = -ku\sqrt{u^2+v^2}\,/(h+\zeta)\,,$$
$$v_t + uv_x + vv_y + g\zeta_y = -kv\sqrt{u^2+v^2}\,/(h+\zeta)\,, \qquad (1)$$
$$\zeta_t + [(h+\zeta)u]_x + [(h+\zeta)v]_y = 0,$$

where $u = u(x, y, t)$ and $v = v(x, y, t)$ are the depth-averaged projections of the horizontal velocity on the axis x and y respectively; t is the time; g is the free fall acceleration; $\zeta = \zeta(x, y, t)$ is the fluid free surface displacement; $h(x, y)$ is the unperturbed fluid depth; $k = 2.6 \cdot 10^{-3}$ is the quadratic bottom friction coefficient.

In the start time the fluid in the bay is in a perturbed state:

$$u(x,y,0) = v(x,y,0) = \zeta(x,y,0) = 0\,. \qquad (2)$$

Through the open boundary $(0 \le x \le L, y = 0)$ a plane wave of half-sine shape enters the bay. It is simulated by the following boundary conditions

$$\zeta = a_0 \sin(\pi C t/\lambda),\quad v = g/C\zeta \quad (0 \le x \le L, y = 0,\ 0 \le t \le \lambda/C)\,, \qquad (3)$$

where $C(x,0) = \sqrt{gh(x,0)}$ is the local velocity of linear long wave propagation.

After a complete entrance of the waves to the bay the free wave entry condition is taken on the liquid boundary

$$v_t - Cv_y = 0 \;\; (0 \le x \le L, y = 0,\ t \ge \lambda/C)\,. \qquad (4)$$

The fluid impermeability condition is taken on the solid coastal boundary

$$u = 0,\quad v = 0. \qquad (5)$$

The initial wave height a_0 is taken to be 1 m.

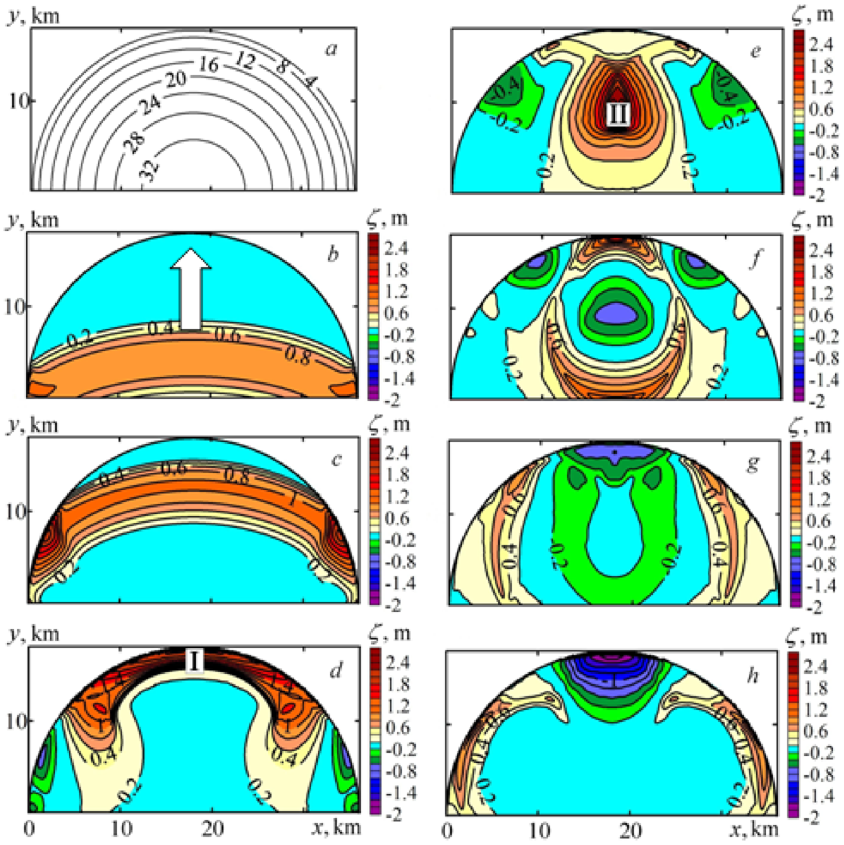

Fig. 1. Transformation of nonlinear long wave propagating in the bay of model geometry: bottom topography (*a*) and the structure of the wave field at the following time moments 500 s (*b*), 1000 s (*c*), 1500 s (*d*), 2100 s (*e*), 2500 s (*f*), 2900 s (*g*), 3200 s (*h*). The wavelength at the bay entrance is 10 km

Propagation of the nonlinear long waves in the bays of model geometry. The shallow bay bounded by semicircular coastline is under consideration (Fig. 1, *a*). Then the features of the nonlinear long waves in this bay are studied. In the numerical simulation a uniform grid with spatial steps $\Delta x = \Delta y = 100$ m and time step $\Delta t = 1$ s is applied.

The plane 10 km wave enters the aforementioned bay through the open boundary (arrows indicate the direction of the wave propagation). There is a bend of its front, as seen in Fig. 1, *b*. As it propagates in the bay two elevations in the shallow areas along the coastline appear (Fig. 1, *c*). Near positive level waves clearly traceable troughs are formed. The wave height increases considerably under its reflection from the shore (Fig. 1, *d*). Here, the first zone of amplification with the wave height of 2 m is situated (I). Having reflected from the coast, the wave forms a local rise of sea level in the middle part, concentrating there the most of its energy (Fig. 1, *e*). Here, the second zone of wave amplification in a bay (II) is found. Near this zone the depths are more significant when compared with the depths of the first wave amplification zone; wave height can be up to 3 m or more. After lowering of the elevation the annular wave directed to the bay exit is formed

(Fig. 1, *f*). In the area of the second maximum followed the positive level wave the sea surface is lowered (Fig. 1, *f*). As for the two troughs, propagating along the coastline, they form the sea level fall in the initial run-up zone on the coast (Fig. 1, *g*, *h*). With the course of time, the wave leaves the bay water area completely.

Fig. 2 shows the distribution of sea-level under the propagation of the waves in the bays with different entrance width at the time of the wave reflection from the coastline (Fig. 2, *a*, *b* and *c*) and when a local maximum in the middle of the bay appear (Fig. 2, *d*, *e* and *f*). In the bay with partially limited entrance the wave is emitted as the annular one and its front repeats in the form the outline of the shore. With the reduction of the bay entrance width the local maximum wave height is also reduced. Approaching the coast, the wave amplifies and its length decreases. The position of the second maximum, which is formed in the middle of the bay, under the entrance narrowing is shifted towards it. There are no significant falls of sea level observed in this bay.

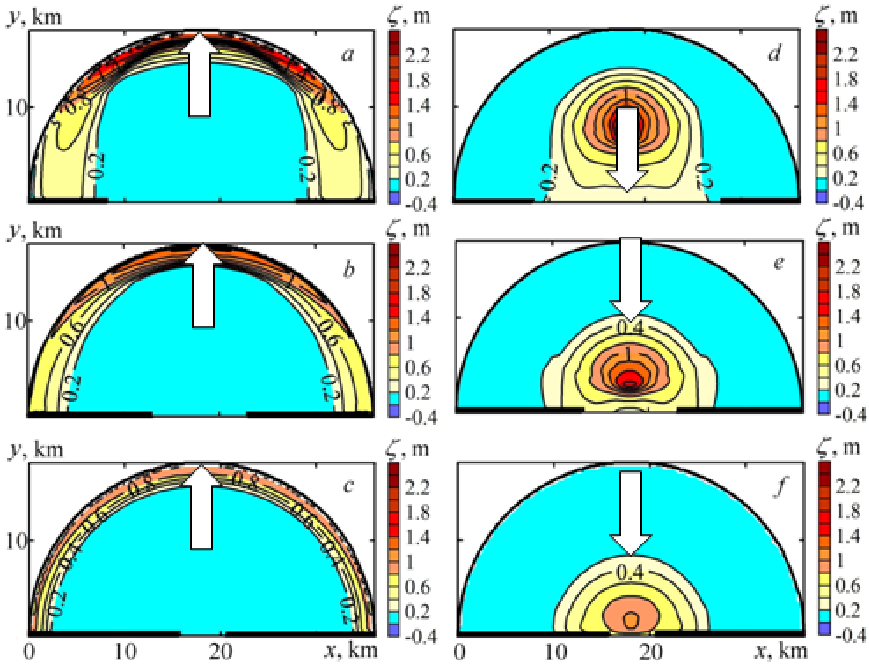

Fig. 2. Propagation of the nonlinear long wave in the bays with different entrance widths: *a*, *b*, *c* are the moments of the maximum run-up of the wave on the lateral boundary of the bays with the entrance width of 20, 10 and 5 km respectively; *d*, *e*, *f* in the moment of the wave going out of the bays

Thus, there are two local areas of the strongest wave amplification in the bay. Below we are to estimate the height of the wave propagating in the bays with different entrance widths. Fig. 3 shows the dependence of the maximum sea level rise from the initial wavelength for the first (along the coastline) (Fig. 3, *a*) and the second (in the middle of the bay) (Fig. 3, *b*) local maxima, respectively.

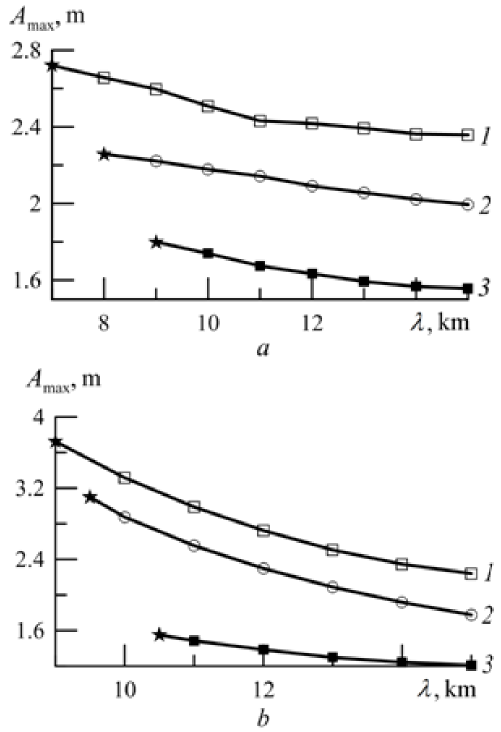

Fig. 3. Maximal rises of sea level in relation from the initial wavelength in the zones of the largest local amplification: *a* – in the zone I; *b* – in the zone II. Foe curves *1, 2, 3* the bay entrance width is 40, 20 and 10 km respectively. Asterisk separates the modes of the breaking (to its left) and non-breaking waves (to its right) waves

Dependencies of the largest falls of sea level on the length of the wave entering the bay are shown in Fig. 4. The amplitude of the wave decreases with increasing of its length. It is known that the inclusion of a quadratic bottom friction in the equations of motion leads to a stronger decrease in the amplitude of the wave as the depth of the bay is smaller. Also, numerical computations showed that the inclusion of bottom friction could increase the distance the wave passes till the break.

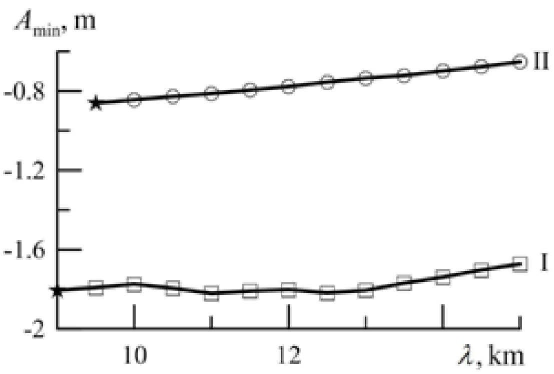

Fig. 4. Extreme falls of sea level in the zones I and II in relation from the initial wavelength

Propagation of the long waves in the Feodosiya Gulf. The Feodosiya Gulf is located in the southeast of the Crimean Black Sea coast. Its depth at the entrance is 20 – 30 m. Fig. 5 shows the distribution of depths in the Feodosiya Gulf. The minimum depth near the coast was taken to be 4 m. The shape and bathymetry of the Gulf resemble the previously considered bay of model geometry (Fig. 1, *a*).

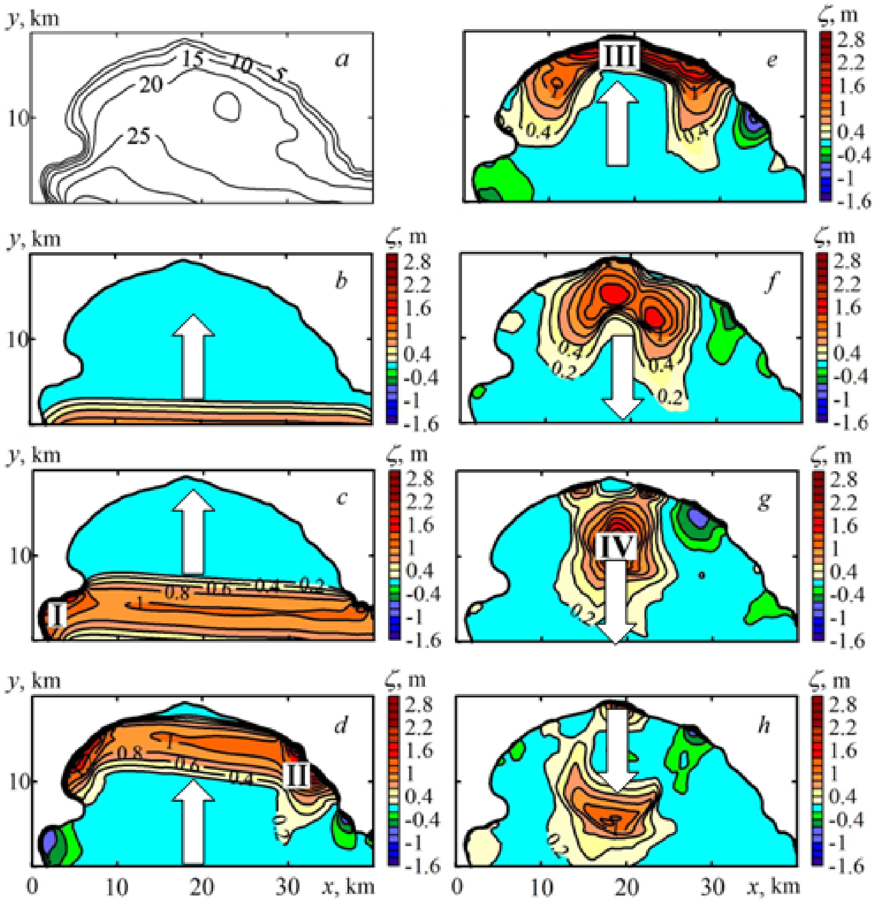

Fig. 5. Propagation of the long waves in the Feodosiya Gulf in the Black Sea: the bottom topography (*a*) and distribution of the sea level displacements in the following instants of time: 200 s (*b*), 610 s (*c*), 1200 s (*d*), 1400 s (*e*), 1550 s (*f*), 2030 s (*g*), 2300 s (*h*). Wavelength at the bay entrance is 10 km

Below we are to consider the propagation of nonlinear solitary long wave in the Feodosiya Gulf. Its initial length is 10 km. Wave enters the Gulf. Its front curves (Fig. 5, *b*, arrows indicate the direction of wave propagation). There are two elevations (Fig. 5, *c*) along the coastline. On the left the wave amplifies more considerably due to a sharp change of the depth and in the presence of the protruding cape. The wave height here reaches 2 – 3 m. This is the first zone of local wave amplification in the bay (I).

As the wave propagates, its height increases and reflecting from the coast reaches up to 3 m (Fig. 5, *d*, *e*). Along the coastline two zones of the highest local wave amplification are marked (II, III). The reflected wave amplifies in the middle part of the bay (Fig. 5, *g*), where there is another local maximum (IV), propagating to the bay exit (Fig. 5, *h*).

Also the evaluations of extreme rises and falls of sea level, depending on the length of the waves entering the bay, were carried out (Fig. 6). In the amplification zones I, II and IV the wave height can reach 3 meters. The most dangerous was the zone III, located at the coastline of the Gulf. The maximum height of the waves here can reach 4 m. And when the wavelength decreases, its maximum height increases. It should be also noted that the measured quadratic bottom friction led to an increase in the distance traversed by the wave to break and reduce its wave height.

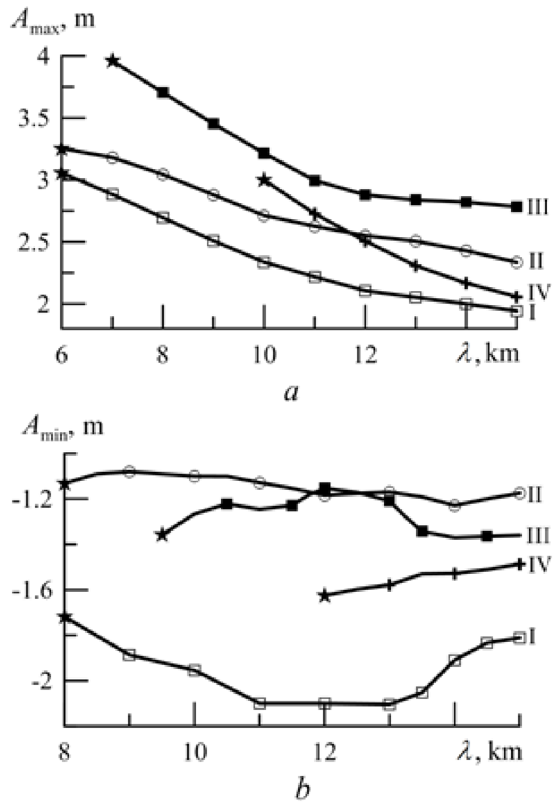

Fig. 6. Dependencies of the maximum sea level displacements in the Feodosiya Gulf on the initial wavelength in the zones of the highest local amplification: *a* – maximum elevation; *b* – the extreme falls of sea level. Asterisk separates the modes of the breaking (to its left) and non-breaking waves (to its right) waves

When the waves in the Feodosiya Gulf propagate in the mentioned amplification zones, troughs following the positive level waves are observed. The following extreme falls of sea level in the Gulf (Fig. 6): in the first zone they reach – 2 m, in the remaining three zones the maximum falls range from –1 to –1.6 m depending on the length of the waves entering the Gulf.

Conclusions. Within the framework of the nonlinear theory of long waves the problem on the propagation of solitary long waves in the semi-closed bays of model and real geometry has been solved. It was established that in the bay with a wide entrance a solitary plane wave propagates forming the local rises and falls of the sea level. In the semi-enclosed bay, it is amplified as an annular one without the considerable fall of the sea level. In the bays having different entrance widths the following two most dangerous wave amplification zones are revealed: the zone of wave reflection from the shore and the one of its amplification in the middle of the bay. In the open bay in the local amplification zones the significant rises and falls of sea level are observed, while in the bay with a partially restricted entrance positive level waves are most expressed. In the second zone of local amplification the greatest heights of all the wave propagation period are monitored. When the bay entrance is narrowing, the maximum wave height considerably reduces.

In general, their dynamics in the Gulf is like the dynamics of waves in the previously reviewed bay of model geometry. In the Feodosiya Gulf the four most dangerous zones of local wave amplification are revealed. There extreme rises and falls of sea level are observed. Numerical computations showed that the maximum height of the waves caused by the reflection from the shore can reach more than 3 meters, and extreme falls occur in the western part of the bay, near the protruding cape, and can reach 2 m.

Thus, the maximum heights of the waves increase with the decrease length of the waves entering the bay. The distance, covered by the wave before the break (due to non-linear effects), is reduced with the decrease of length of the wave entering the bay.

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