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published in MORSKOY GIDROFIZICHESKIY ZHURNAL, Vol. 38, Iss. 4 (2022)

Vertical Transfer of Momentum by Internal Waves in the Western Part of the Mediterranean Sea

A. A. Slepyshev ✉, A. V. Nosova

Marine Hydrophysical Institute of RAS, Sevastopol, Russian Federation
✉ *slep55@mail.ru*

Abstract
Purpose. The work is aimed at studying both the momentum vertical transfer by internal waves with the regard for the Earth rotation and the shear current in the western part of the Mediterranean Sea, and the influence of turbulent viscosity and diffusion upon the indicated wave fluxes and the Stokes drift.
Methods and Results. To solve the initial system of the hydrodynamics nonlinear equations, a weakly nonlinear approach was used. In the first order of smallness in the wave amplitude, the boundary problem for the vertical velocity amplitude was solved; in the second order in the wave amplitude, the nonlinear effects, namely the Stokes drift and the vertical wave momentum fluxes, were investigated. The indicated boundary problem was solved in two ways: by the perturbation method applied earlier and by the numerical one by the implicit Adams scheme of the third order of accuracy. The perturbation method assumes expansion of the solution and the wave frequency in a series by a small parameter proportional to the horizontal turbulent viscosity. The results obtained by the perturbation and numerical methods were compared. Coincidence of the results of calculating the dispersion curves by both methods is shown. However, for the wave damping decrement, the perturbation method yields the overestimated values, at that for the second mode the values are higher than those for the first one. The vertical wave momentum fluxes are nonzero, and the perturbation method yields the overestimated values for the flux $\overline{uw}$. The vertical wave momentum flux $\overline{vw}$ is practically independent of turbulent viscosity and diffusion, and both methods give the identical results for it. The velocity component of the Stokes drift along the wave propagation direction is also insensitive to turbulent viscosity and diffusion, whereas the transverse component equals zero in the absence of turbulent viscosity and diffusion.
Conclusions. The perturbation method provides the overestimated values of the wave damping decrements, the wave momentum flux $\overline{uw}$ and the transverse component of the Stokes drift velocity. The horizontal turbulent viscosity and diffusion exert practically no effect upon the wave momentum flux $\overline{vw}$ and the longitudinal component of the Stokes drift velocity.

Key words: internal waves, wave momentum flux, Stokes drift

Acknowledgements: the study was carried out within the framework of the state assignment on theme No. 0555-2021-0004.

For citation: Slepyshev, A.A. and Nosova, A.V., 2022. Vertical Transfer of Momentum by Internal Waves in the Western Part of the Mediterranean Sea. *Physical Oceanography*, 29(4), pp. 334-346. doi:10.22449/1573-160X-2022-4-334-346

DOI: 10.22449/1573-160X-2022-4-334-346

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Introduction. Internal waves play an important role in the dynamic processes of the ocean. This is especially true of the shelf and the continental slope. The sources of internal wave generation are different: atmospheric disturbances, interaction of currents and tides with bottom relief inhomogeneities [1], generation of internal waves by synoptic eddies. Internal waves can be generated in the case of the hydrodynamic instability of currents [2]. In this paper, the vertical exchange is

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associated with internal waves, and this is not accidental. Internal waves often generate turbulence, which causes mixing of stratified layers of liquid. Thus, small-scale turbulence causes vertical exchange in the ocean [3]. Turbulence is generated by the breaking of surface and internal waves [3, 4]. The hydrodynamic instability of currents and internal waves also leads to turbulence generation [2, 5, 6]. Hydrodynamic instability of currents often generates internal waves, which also become unstable and generate smaller-scale waves [7]. Consequently, there is a cascade transfer of energy on a small scale, to the point of turbulence. The breaking of internal waves is by no means a typical process in the ocean, much more often turbulence is supported by a weak shift in the flow velocity in the internal wave [1].

In a stratified marine environment, turbulence is strongly suppressed, but due to stratification, internal waves exist, therefore, the study of their contribution to the vertical exchange seems to be an important and urgent task. The problem of internal waves and turbulence interaction is still far from being solved since nonlinear interactions play a key role in it.

Turbulent viscosity and diffusion make it possible to describe the effect of small-scale turbulence on internal waves, which attenuate when taking into account turbulent viscosity and diffusion [8–10]. In this case, the vertical wave momentum fluxes are different from zero [11, 12]. However, these fluxes were found when solving the boundary value problem for the vertical velocity amplitude by the perturbation method by decomposing the solution and the wave frequency into series according to a small parameter proportional to the horizontal turbulent viscosity. In this paper, this boundary value problem is solved numerically by the implicit Adams scheme of the third order of accuracy. It is of interest to compare wave momentum fluxes obtained by the numerical and perturbation methods. The same applies to the velocity of the Stokes drift of liquid particles. The vertical wave momentum fluxes of internal waves are different from zero and in the absence of turbulent viscosity and diffusion in the presence of a current in which the velocity component normal to the wave propagation direction depends on the vertical coordinate [13, 14]. In this regard, it is important to study the influence of horizontal turbulent viscosity and diffusion on the mentioned processes.

Problem statement. In the Boussinesq approximation, free inertia-gravity internal waves on a plane-parallel shear flow, taking into account horizontal turbulent viscosity and diffusion, are considered. Nonlinear hydrodynamic equations for wave disturbances are solved in a weakly nonlinear approximation: in the linear approximation the dispersion properties of internal waves are studied and the wave attenuation decrement is found; in the second-order wave amplitude the vertical wave momentum fluxes and the Stokes drift velocity are found.

Motion equations for wave disturbances have the following form:

[1] Ostrovskii, L.A., Soustova, I.A. and Tsimring, L.Sh., 1981. *Influence of Internal Waves on Small-Scale Turbulence in the Ocean*. Preprint No. 31. Gorky: IAP of the USSR Academy of Sciences, 15 p. (in Russian).

$$\frac{\partial u}{\partial t}+(u+U_0)\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv+w\frac{dU_0}{dz}= \\ =-\frac{\partial P}{\rho_0(0)\partial x}+K\frac{\partial^2 u}{\partial x^2}+K\frac{\partial^2 u}{\partial y^2}, \quad (1)$$

$$\frac{\partial v}{\partial t}+(u+U_0)\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+fu=-\frac{\partial P}{\rho_0(0)\partial y}+K\frac{\partial^2 v}{\partial x^2}+K\frac{\partial^2 v}{\partial y^2}, \quad (2)$$

$$\frac{\partial w}{\partial t}+(u+U_0)\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}= \\ =-\frac{\partial P}{\rho_0(0)\partial z}+K\frac{\partial^2 w}{\partial x^2}+K\frac{\partial^2 w}{\partial y^2}-g\frac{\rho}{\rho_0(0)}, \quad (3)$$

$$\frac{\partial \rho}{\partial t}+(u+U_0)\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}=-w\frac{d\rho_0}{dz}+M\frac{\partial^2 \rho}{\partial x^2}+M\frac{\partial^2 \rho}{\partial y^2}, \quad (4)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \quad (5)$$

where x, y, z are cartesian coordinates, z axis is directed opposite to the free fall acceleration, g; u, v, w are the projection of current wave velocity in the specified coordinate system; ρ_0 is undisturbed average water density; P and ρ are wave disturbances of pressure and density; f is the Coriolis parameter; K, M are the coefficients of horizontal turbulent viscosity and diffusion, which are assumed to be constant; $U_0(z)$ is a mean current velocity. Let us make an estimate of the horizontal scale of the average density variation. For this purpose, geostrophic ratios [2] are to be used. An estimate for the specified scale $L_\rho=\frac{\rho_0}{|\partial\rho_0/\partial y|}=\frac{g}{f\,|\partial U_0/\partial z|}$ follows from them. The vertical gradient profile of the average current velocity is demonstrated in Fig. 1, *a*. The maximum value of the modulus of the current velocity vertical gradient is $2.2\cdot10^{-2}\cdot$ 1/s, then L_ρ is at least 5.19×10^6 m, i.e., much longer than the wavelength. Thus, the dependence of the average density on the horizontal coordinate can be neglected.

We use the “rigid lid” conditions on the surface and the absence of tangential stresses as boundary conditions [3] [9]:

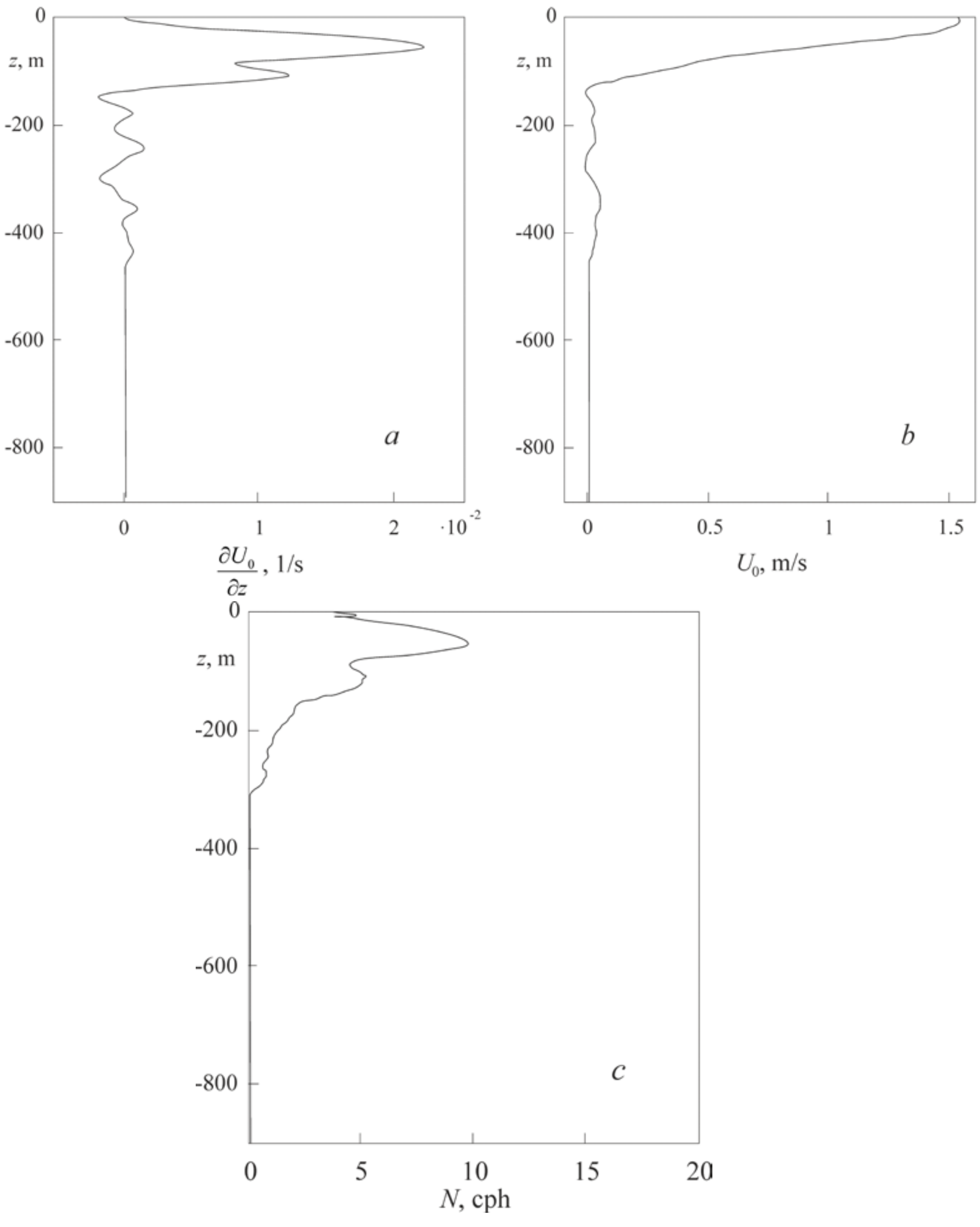
$$w=0, \qquad K\frac{\partial w}{\partial x}=0, \qquad K\frac{\partial w}{\partial y}=0, \qquad z=0, \quad (6)$$

at the bottom, there is an impermeability condition and also the absence of tangential stresses [3] [9]:

[2] Kamenkovich, V.M., 1977. *Fundamentals of Ocean Dynamics.* Amsterdam; New York: Elsevier Scientific Pub. Co., 240 p.
[3] Miropol'sky, Yu.Z., 2001. *Dynamics of Internal Gravity Waves in the Ocean.* Dordrecht: Springer, 406 p.

$$w=0, \quad K\frac{\partial w}{\partial x}=0, \quad K\frac{\partial w}{\partial y}=0, \qquad z=-H, \tag{7}$$

where H is sea depth.



F i g. 1. Vertical profiles of the current velocity gradient (*a*), the current velocity (*b*) and the Brunt–Väisälä frequency (*c*)

At the bottom, the tangential stresses are equal to zero, since the vertical exchange coefficients are neglected in comparison with the horizontal turbulent exchange coefficients.

Linear approximation. The linear approximation solutions can be found in the following form [11–14]:

$$u = u_{10}(z)A\exp(i(kx-\omega t)) + \text{c.c.},\ v = v_{10}(z)A\exp(i(kx-\omega t)) + \text{c.c.},$$
$$w_1 = w_{10}(z)A\exp(i(kx-\omega t)) + \text{c.c.},$$
$$P_1 = P_{10}(z)A\exp(i(kx-\omega t)) + \text{c.c.},\ \rho_1 = \rho_{10}(z)A\exp(i(kx-\omega t)) + \text{c.c.}, \qquad (8)$$

where A is an amplitude multiplier; k is a horizontal wave number; ω is a wave frequency; c.c. are complex conjugate terms.

After substituting (8) into the system (1) – (5), the connection of the amplitude functions $u_{10}, P_{10}, \rho_{10}, v_{10}$ with w_{10} follows [11, 12]:

$$u_{10} = \frac{i}{k}\frac{dw_{10}}{dz}, \qquad \Omega = \omega - k\cdot U_0, \qquad (9)$$

$$\frac{P_{10}}{\rho_0(0)} = \frac{i}{k}\left[\frac{\Omega}{k}\frac{dw_{10}}{dz} + \frac{dU_0}{dz}w_{10} - \frac{if^2}{k\left(i\Omega - k^2K\right)}\frac{dw_{10}}{dz} + ikK\frac{dw_{10}}{dz}\right],$$

$$\rho_{10} = \frac{w_{10}}{i\Omega - k^2M}\frac{d\rho_0}{dz}, \qquad v_{10} = \frac{if}{k\left(i\Omega - k^2K\right)}\frac{dw_{10}}{dz}. \qquad (10)$$

Function w_{10} satisfies the following equation [11, 12]:

$$(\Omega + ik^2K)\left[\Omega^2 + 2i\Omega k^2K - f^2 - k^4K^2\right]\frac{d^2w_{10}}{dz^2} - kf^2\frac{dU_0}{dz}\frac{dw_{10}}{dz} +$$
$$+k\left[\left(\frac{d^2U_0}{dz^2} - k\Omega - ik^3K\right)\left(\Omega + ik^2K\right)^2 + kN^2\frac{(\Omega + ik^2K)^2}{(\Omega + ik^2M)}\right]w_{10} = 0, \qquad (11)$$

where $N^2 = -\frac{g}{\rho_0(0)}\frac{d\rho_0}{dz}$ is a square of the Brunt–Väisälä frequency.

Boundary conditions have the following form:

$$w_{10}(0) = w_{10}(-H) = 0. \qquad (12)$$

The remaining boundary conditions in (6), (7) are fulfilled automatically.

The boundary value problem (11), (12) in [11, 12] was solved by the perturbation method, when the solution and the frequency of the wave were decomposed in a series by a small parameter proportional to the horizontal turbulent viscosity value. In this paper, this problem is solved numerically by the implicit Adams scheme of the third order of accuracy. A comparison of the dispersion curves and the dependence of the wave attenuation decrement on the frequency for the first two modes is given below.

Nonlinear effects. The relationship between the current velocity in the Euler $\mathbf{u}$ and Lagrange $\mathbf{u}_L$ representations with accuracy up to the terms of the second order in the steepness of the wave has the following form [4] [15]:

$$\mathbf{u}_L = \mathbf{u} + \left(\int_0^t \mathbf{u}_L d\tau \nabla\right)\mathbf{u}. \quad (13)$$

This integral equation is solved by the iterative method. With an accuracy of the terms, quadratic in the wave amplitude, after averaging over the wave period, an expression for the average Lagrangian velocity is obtained [4]:

$$\overline{\mathbf{u}_L} = \mathbf{U} + \overline{\left(\int_0^t \mathbf{u} d\tau \nabla\right)\mathbf{u}}, \quad (14)$$

where vector $\mathbf{U}(U_0, V_0)$ is a mean current velocity (V_0 = 0); $\mathbf{u}$ is a field of Euler velocities, a line above means averaging over the wave period. Velocity of the Stokes drift of liquid particles is represented by the second term in (14) and is determined by the following formula [4] [15]:

$$\mathbf{u}_s = \overline{\left(\int_0^t \mathbf{u} d\tau \nabla\right)\mathbf{u}}. \quad (15)$$

The components of the Stokes drift velocity along and across the wave propagation direction have the following form [11, 14]:

$$u_s = \frac{A_1 A_1^*}{k}\left[\frac{1}{\omega}\frac{d}{dz}\left(w_{10}\frac{dw_{10}^*}{dz}\right) + \text{c.c.}\right], \quad (16)$$

$$v_s = A_1 A_1^* \frac{i}{\omega}\frac{d}{dz}\left[\frac{w_{10}}{\left(\Omega^* - ik^2 K\right)}\left(\frac{f}{k}\frac{dw_{10}^*}{dz}\right)\right] + \text{c.c.}, \quad (17)$$

where $A_1 = A\exp(\delta\omega \cdot t)$, $\delta\omega = \text{Im}(\omega)$ is the imaginary part of the frequency, the wave attenuation decrement.

From (8) – (10), the expressions for vertical wave momentum fluxes $\overline{uw}$, $\overline{vw}$ follow [11, 14]:

$$\overline{uw} = \frac{i}{k}\left|A_1^2\right|\left(w_{10}^*\frac{dw_{10}}{dz} - w_{10}\frac{dw_{10}^*}{dz}\right), \quad (18)$$

$$\overline{vw} = \frac{w_{10}^*\left|A_1^2\right|}{\left(\Omega + ik^2 K\right)}\left(\frac{f}{k}\frac{dw_{10}}{dz}\right) + \text{c.c.} \quad (19)$$

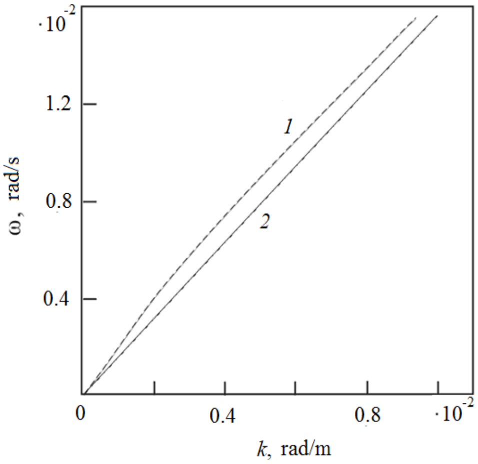
[4] Dvoryaninov, G.S., 1982. [*Effects of Waves in the Boundary Layers of the Atmosphere and Ocean*]. Kiev: Naukova Dumka, 25 p. (in Russian).

In inertia-gravity internal waves, the momentum flux $\overline{vw}$ is nonzero. The momentum flux $\overline{uw}$ is different from zero only in the presence of turbulent viscosity and diffusion, in this case it is not equal to zero even if the Earth rotation is not taken into account. The component of the Stokes drift velocity transverse to the wave propagation direction is different from zero for inertia-gravity internal waves only when turbulent viscosity and diffusion are taken into account, if the rotation of the Earth is not taken into account, then it is zero.

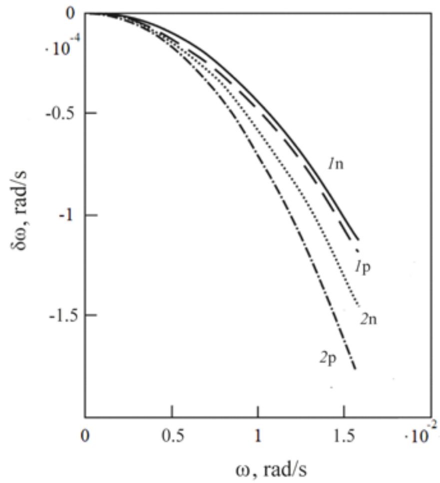
Results of calculations. The Strait of Gibraltar connects the Mediterranean Sea with the Atlantic Ocean. The Atlantic Ocean waters, less salty, enter the Strait of Gibraltar at high velocity, being located in the upper 100-meter layer. The saltier waters, which flow more slowly from the Mediterranean Sea, are deeper. At the exit from the Strait of Gibraltar to the Mediterranean Sea, a layer with a sharp drop in current velocity and water density is formed, the corresponding vertical profiles of current velocity and stratification are demonstrated in Fig. 1, *b*, *c* [16]. The semi-daily tide, running from the Atlantic Ocean to the Strait of Gibraltar, generates internal waves. In particular, during a full-scale experiment, powerful wave trains of the lowest mode of internal waves with 14 min period were detected [16]. In order to clarify the vertical structure of internal waves, the boundary value problem (11), (12) is solved numerically according to the implicit Adams scheme of the third order of accuracy at $K = 2M$. At the considered scales, the typical value of the horizontal turbulent diffusion coefficient is $M = 1$ m^2/s [17]. For a fixed wave frequency, the wave number and the wave attenuation decrement are determined by the shooting method. The results of numerical calculations are compared with the results of solving the boundary value problem (11), (12) by the perturbation method [12], when the solution and the frequency of the wave are searched for as a series for a small parameter proportional to the horizontal turbulent viscosity.

From Fig. 2 it follows that the dispersion curves constructed by the perturbation method and by numerical solution of the boundary value problem (11), (12) actually coincide. Fig. 3 demonstrates the dependence of the imaginary part of the wave frequency on the real part of the frequency for the first (*1*) and second (*2*) modes calculated by the perturbation method (curves *1*p, *2*p) and by numerical solution of the boundary value problem (11), (12) (curves *1*n, *2*n). The perturbation method provides overestimated values of the module of the wave frequency imaginary part, for the second mode – larger ones.

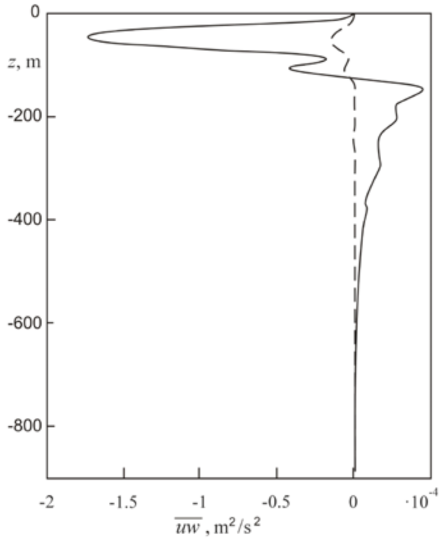
The normalizing multiplier A_1 is found by the known value of the maximum amplitude of vertical displacements [12]. The vertical wave momentum fluxes $\overline{uw}$ (formula (18)) for 14-minute internal waves of the first mode with 16 m amplitude, obtained both in the numerical solution of the boundary value problem (11), (12), and when using the perturbation method, are given in Fig. 4. The perturbation method provides overestimated values of the vertical momentum flux $\overline{uw}$.



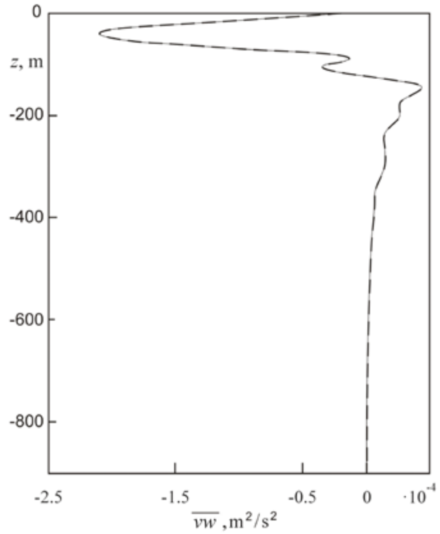
F i g. 2. Dispersion curves of the first (*1*) and second (*2*) modes of internal waves



F i g. 3. Dependence of the damping decrement upon the wave frequency obtained by the perturbation method (curves *1*p, *2*p) and by the numerical solution of the boundary problem (curves *1*n, *2*n)



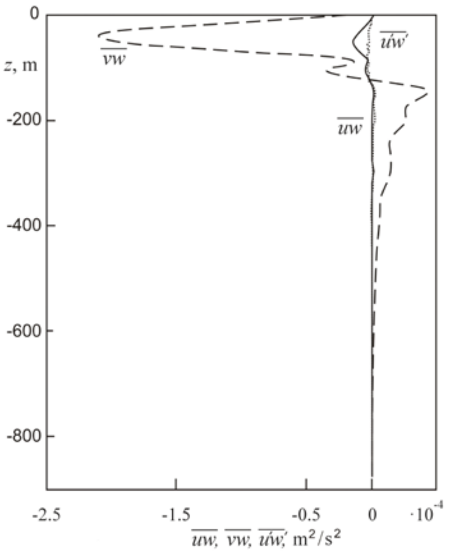
F i g. 4. Profiles of the vertical wave flux of momentum $\overline{uw}$ obtained by the perturbation method (solid line) and by the numerical one (dashed line)



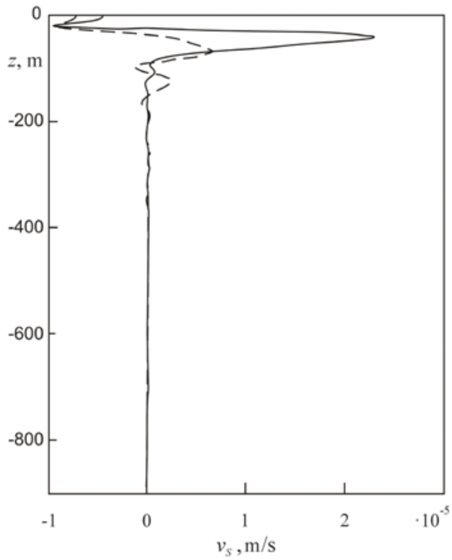
F i g. 5. Profiles of the vertical wave momentum flux $\overline{vw}$

Vertical wave momentum flux $\overline{vw}$ (formula (19)) is shown in Fig. 5. The perturbation and numerical methods give identical results, moreover, the wave flux $\overline{vw}$ is practically not affected by horizontal turbulent viscosity and diffusion.

The wave vertical momentum fluxes $\overline{uw}$, $\overline{vw}$ and the corresponding turbulent flux $\overline{u'w'}$ are compared in Fig. 6. The turbulent momentum flux is determined by the gradient hypothesis: $\overline{u'w'} = -K_z \frac{dU_0}{dz}$. The vertical exchange coefficient in the upper 150-meter highly stratified layer is inversely proportional to the Brunt–Väisälä frequency: $K_z = 8.4 N_c^{-1} \cdot 10^{-4}$ m²/s, here N_c corresponds to the Brunt–Väisälä frequency (cph) [18]. Deeper, where the stratification is weak, the coefficient of turbulent exchange is proportional to N_c [19]. In general, the wave momentum fluxes are superior to the turbulent one in modulus. The wave flux $\overline{vw}$ exceeds the $\overline{uw}$ flux in absolute magnitude.



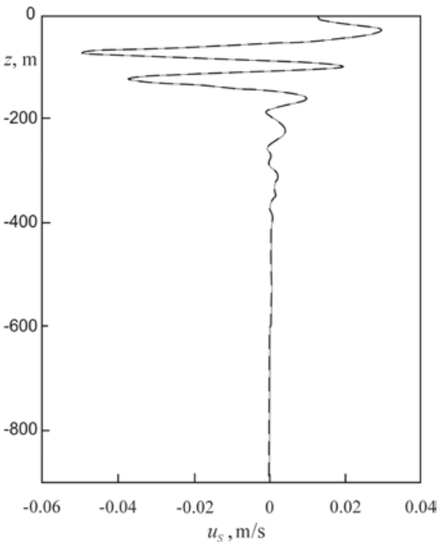
F i g. 6. Profiles of the wave (solid and dashed lines) and turbulent $\overline{u'w'}$ (dotted line) vertical momentum fluxes



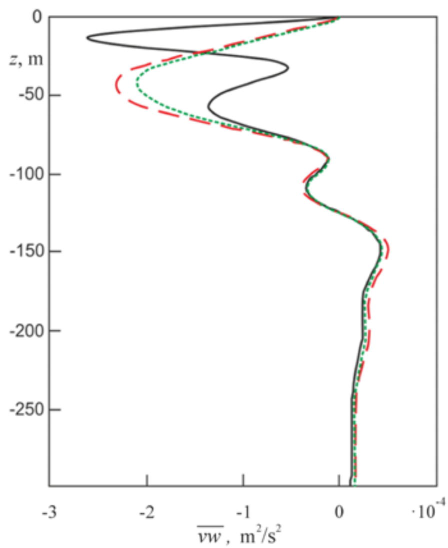
F i g. 7. Vertical profiles of transverse component of the Stokes drift velocity obtained by the perturbation method (solid line) and by the numerical solution of the boundary problem (11), (12) (dashed line)

The vertical profiles of the Stokes drift velocity component v_s transverse to the wave propagation direction, calculated by the perturbation method and the numerical method, are shown in Fig. 7. The perturbation method provides overestimated values in the upper 70-meter layer. In the absence of turbulent viscosity and diffusion $v_s = 0$.

The longitudinal component of the Stokes drift velocity u_s is demonstrated in Fig. 8. The perturbation method and numerical methods give identical results, moreover, turbulent viscosity and diffusion having practically no effect on this component of the Stokes drift velocity. The longitudinal component of the Stokes drift velocity is three orders of magnitude greater than the transverse one.



F i g. 8. Vertical profiles of the longitudinal component of the Stokes drift velocity

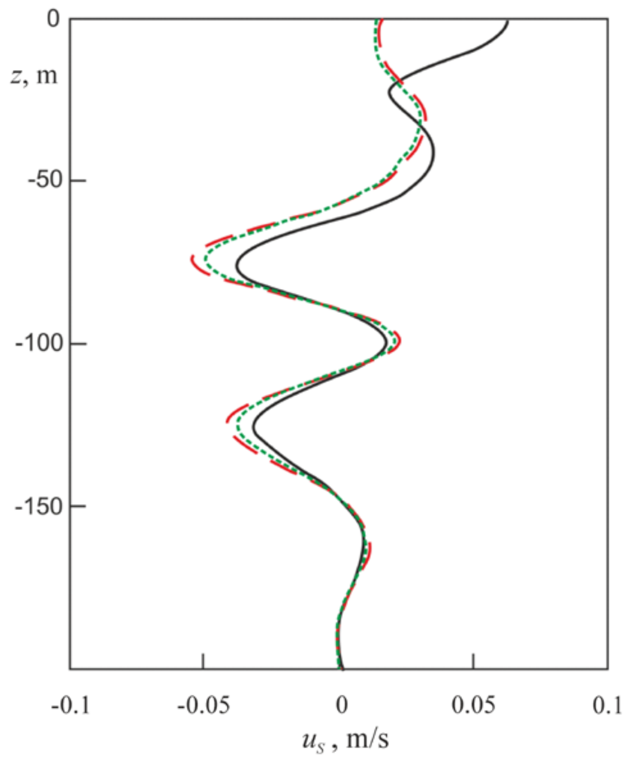


F i g. 9. Profiles of the vertical wave momentum flux $\overline{vw}$ at the increased turbulent exchange coefficients obtained by the perturbation method (black curve), by the numerical one (red curve) and in the inviscid case (green curve)

It is still of interest to study the effect of turbulent viscosity and diffusion on the wave momentum flux $\overline{vw}$ and on the Stokes drift velocity u_s at large coefficients of turbulent exchange. Fig. 9 shows the profiles of vertical wave momentum flux $\overline{vw}$ calculated by the perturbation and numerical methods at $M = 50$ m^2/s, as well as for the inviscid case at $M = K = 0$. The perturbation method provides strong distortions of the vertical momentum flux in the upper 70-meter layer. The wave momentum flux $\overline{vw}$, according to numerical calculations, is slightly larger than the flux with no regard to turbulent viscosity and diffusion.

Fig. 10 demonstrates the vertical profiles of the Stokes drift velocity u_s of the first-mode 14-minute internal waves, calculated by the perturbation method, the numerical method and for the inviscid case at $M = K = 0$. The perturbation method provides a noticeable distortion of the Stokes drift velocity in the upper 25-meter layer.

Numerical calculation with regard to turbulent viscosity and diffusion gives slightly larger values of the Stokes drift velocity u_s in modulus compared to the inviscid case when turbulent viscosity and diffusion are not taken into account.



F i g. 10. Vertical profiles of the Stokes drift velocity u_s at the increased turbulent exchange coefficients obtained by the perturbation method (black curve), by the numerical one (red line) and in the inviscid case (green curve)

Conclusions

1. When solving the boundary value problem (11), (12) by the perturbation method, the vertical wave momentum flux $\overline{uw}$ exceeds the flux $\overline{uw}$ when solving this problem by the numerical method according to the implicit Adams scheme of the third order of accuracy.

2. The perturbation and numerical methods provide identical results for the vertical wave momentum flux $\overline{vw}$, and taking into account horizontal turbulent viscosity and diffusion practically does not change it. Similarly, the of Stokes drift velocity along the wave propagation direction is practically insensitive to regarding for turbulent viscosity and diffusion. If the turbulent exchange coefficients are increased by 50 times, then the perturbation method will give a noticeable error, especially in the upper layer. Numerical calculation in this case shows that taking into account turbulent viscosity and diffusion slightly increases the wave momentum flux $\overline{vw}$ and the Stokes drift velocity u_s.

3. The perturbation method for the transverse component of the Stokes drift velocity gives an overestimate in the upper 70-meter layer, and if the turbulent viscosity and diffusion are not taken into account, then it is equal to zero.

REFERENCES

1. Bulatov, V.V. and Vladimirov, Yu.V., 2015. *Waves in Stratified Medium*. Moscow: Nauka, 735 p. (in Russian).

2. Korotaev, G.K. and Panteleev, N.A., 1977. Experimental Studies of Hydrodynamic Instability in the Ocean. *Okeanologiya*, 17(6), pp. 941-953 (in Russian).

3. Wunsch, C. and Ferrari, R., 2004. Vertical Mixing, Energy, and the General Circulation of the Ocean. *Annual Review of Fluid Mechanics*, 36, pp. 281-314. doi:10.1146/annurev.fluid.36.050802.122121

4. Samodurov, A.S., Lubitsky, A.A. and Panteleev, N.A., 1995. Contribution of Breaking Internal Waves to Structure Formation, Energy Dissipation, and Vertical Diffusion in the Ocean. *Physical Oceanography*, 6(3), pp. 177-190. doi:10.1007/BF02197516

5. Korotaev, G.K. and Panteleev, N.A., 1977. Hydrodynamical Instability of Internal Waves with Nonstationary Shear. *Izvestiya Akademii Nauk SSSR. Fizika Atmosfery i Okeana*, 13(10), pp. 1044-1054 (in Russian).

6. Podymov, O.I., Zatsepin, A.G. and Ostrovsky, A.G., 2017. Vertical Turbulent Exchange in the Black Sea Pycnocline and Its Relation to Water Dynamics. *Oceanology*, 57(4), pp. 492-504. doi:10.1134/S0001437017040142

7. Panteleev, N.A., Shcherbakov, A.N. and Shcherbakova, E.N., 1989. The Observation of the Hydrodynamical Instability in the Ocean. *Izvestiya Akademii Nauk SSSR. Fizika Atmosfery i Okeana*, 25(6), pp. 616-626 (in Russian).

8. LeBlond, P.H. and Mysak, L.A., 1978. *Waves in the Ocean*. Amsterdam–Oxford–New York: Elsevier Scientific Publishing Company, 602 p.

9. LeBlond, P.H., 1966. On the Damping of Internal Gravity Waves in a Continuously Stratified Ocean. *Journal of Fluid Mechanics*, 25(1), pp. 121-142. doi:10.1017/S0022112066000089

10. Ostrovsky, L.A. and Soustova, I.A., 1979. The Upper Mixed Layer of the Ocean as an Energy Sink of Internal Waves. *Okeanologiya*, 19(6), pp. 973-981 (in Russian).

11. Slepyshev, A.A., 2016. Vertical Momentum Transfer by Internal Waves when Eddy Viscosity and Diffusion are Taken into Account. *Izvestiya, Atmospheric and Oceanic Physics*, 52(3), pp. 301-308. doi:10.1134/S0001433816030117

12. Slepyshev, A.A. and Nosova, A.V., 2020. Generation of Vertical Fine Structure by the Internal Waves with the Regard for Turbulent Viscosity and Diffusion. *Physical Oceanography*, 27(1), pp. 3-17. doi:10.22449/1573-160X-2020-1-3-17

13. Ankudinov, N.O. and Slepyshev, A.A., 2021. Vertical Momentum Transfer Induced by Internal Waves in a Two-Dimensional Flow. *Fluid Dynamics*, 56(3), pp. 343-352. doi:10.1134/S0015462821030022

14. Vorotnikov, D.I. and Slepyshev, A.A., 2018. Vertical Momentum Fluxes Induced by Weakly Nonlinear Internal Waves on the Shelf. *Fluid Dynamics*, 53(1), pp. 21-33. doi:10.1134/S0015462818010160

15. Longuet-Higgins, M.S., 1969. On the Transport of Mass by Time-Varying Ocean Currents. *Deep Sea Research and Oceanographic Abstracts*, 16(5), pp. 431-447. doi:10.1016/0011-7471(69)90031-X

16. Watson, G., 1994. Internal Waves in a Stratified Shear Flow: The Strait of Gibraltar. *Journal of Physical Oceanography*, 24(2), pp. 509-517. doi:10.1175/1520-0485(1994)024<0509:IWIASS>2.0.CO;2

17. Bowden, K.F., 1983. *Physical Oceanography of Coastal Waters*. New York: Halsted Press, 302 p.

18. Ivanov, V.A., Samodurov, A.S., Chukharev, A.M. and Nosova, A.V., 2008. Intensification of Vertical Turbulent Exchange in the Vicinity of the Interface between the Shelf and the Continental Slope in the Black Sea. *Reports of the National Academy of Sciences of Ukraine*, (6), pp. 108-112 (in Russian).

19. Samodurov, A.S., 2016. Complimentarity of Different Approaches for Assessing Vertical Turbulent Exchange Intensity in Natural Stratified Basins. *Physical Oceanography*, (6), pp. 32-42. doi:10.22449/1573-160X-2016-6-32-42

About the authors:

Aleksandr A. Slepyshev, Leading Research Associate, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., Sevastopol, 299011, Russian Federation), Dr. Sci. (Phys.-Math.), **ResearcherID: V-6948-2017, ORCID ID: 0000-0002-9259-7558**, slep55@mail.ru

Anna V. Nosova, Leading Engineer-Researcher, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., Sevastopol, 299011, Russian Federation)

Contribution of the co-authors:

Aleksandr A. Slepyshev – theoretical part and obtaining all the formulas

Anna V. Nosova – calculation part, numerical solution of boundary value problems, plotting

The authors have read and approved the final manuscript.

The authors declare that they have no conflict of interest.