Original Russian Text © A. S. Zapevalov, 2023,
published in MORSKOY GIDROFIZICHESKIY ZHURNAL, Vol. 39, Iss. 2 (2023)

Original article

Impact of the Sea Waves' Skewness and Group Structure on the Infrasound Generation by the Sea Surface

A. S. Zapevalov

Marine Hydrophysical Institute of RAS, Sevastopol, Russian Federation
✉ *sevzepter@mail.ru*

Abstract
Purpose. The study is aimed at analyzing the impact of the effects of the sea waves' nonlinearity manifested in the skewness of sea surface elevations and in arising of a group structure, upon the generation of infrasound radiation by the sea surface.
Methods and Results. The analysis is based on the analytical model of a wave profile which permits to set an asymmetric wave profile (a pointed crest and a flat trough), and also to vary the grouping factor and the number of waves in a group. The field of surface waves is represented as a superposition of free waves and harmonics. It was studied using the mathematical apparatus of decomposing the analyzed function into the Fourier series. Quantitative estimates characterizing (in different situations) the ratio between the amplitudes of the acoustic waves generated by the main wave and its harmonics were obtained. It was shown that skewness affected the level of infrasound generation to a greater extent than the group structure of waves.
Conclusions. Both the skewness of sea wave elevations and their group structure lead to a decrease in the level of infrasound generated by the sea surface, as well as to the redistribution of infrasound energy over the spatial and temporal scales.

Keywords: sea surface, free waves, bonded waves, hydroacoustics, infrasound, group structure

Acknowledgments: The investigation was carried out within the framework of the state task on theme No. 0555-2021-0004 "Fundamental studies of the oceanological processes which determine the state and evolution of the marine environment influenced by natural and anthropogenic factors, based on the observation and modeling methods".

For citation: Zapevalov, A.S., 2023. Impact of the Sea Waves' Skewness and Group Structure on the Infrasound Generation by the Sea Surface. *Physical Oceanography*, 30(2), pp. 160-170. doi:10.29039/1573-160X-2023-2-160-170

DOI: 10.29039/1573-160X-2023-2-160-170

Introduction

At present, research that study generation of infrasound by the sea surface are of great scientific and practical interest [1–4]. In 1950, Longuet-Higgins built a model of infrasound generation by the sea surface waves of the same frequency propagating in opposite directions [5]. The infrasonic pressure fluctuations that are generated as a result of the nonlinear interaction of waves, which do not decay with depth, lead to the appearance of microseisms. The existence of a relationship between meteorological conditions and the occurrence of microseisms, which play an important role in geophysical processes, has been revealed [6–8].

The theory of infrasound generation was developed in [9–11], where the models that relate acoustic and spatial spectra of surface waves were constructed. The infrasound generation process is described in terms of the three-wave interaction of two surface waves and one acoustic wave. The main nonlinearity is associated with setting boundary conditions for hydrodynamic equations, while propagation of acoustic waves is described by a linear wave equation [12]. The sea surface is represented as a superposition of harmonic waves, which, by virtue of the central limit theorem, leads to a Gaussian distribution of elevations [13, 14].

In this formulation of the problem, a number of non-linear effects in sea waves is not taken into account. This is a weak nonlinearity caused by wave-wave interaction [15], which, in particular, results in deviations of the distributions of sea surface elevations from the Gaussian distribution [16]. Another manifestation of the nonlinearity of surface waves is their group structure [17, 18], which evolves as a result of the balance between dispersion and nonlinearity [19]. In this paper, we analyze how these factors affect the level of hydroacoustic radiation generated by the sea surface.

Wave profile modeling

The group structure of waves is usually described as the product of a carrier wave $\eta(x,t)$ and its envelope $G(x,t)$. We represent the carrier wave and its envelope in the form proposed in [20]. In this work, the carrier wave has an asymmetric profile, i.e., a pointed crest and a flat trough:

$$\eta(x,t) = \exp\left[-\mu_0 \cos^2\left(\frac{k_0 x - \omega_0 t}{2}\right)\right], \tag{1}$$

where x and t are spatial and temporal coordinates; the parameter μ_0 determines skewness of the carrier wave; k_0 and ω_0 are the wavenumber and cyclic frequency of the carrier wave. The proposed model has a limitation: it does not allow the consideration of a strictly symmetrical carrier wave. To analyze this particular case, it is necessary to specify a harmonic wave instead of (1).

Dominant sea waves, i.e. the waves with a maximum frequency in the wave spectrum, belong to the class of gravitational ones. In the deep-water approximation, when the effect of the bottom can be neglected, the dispersion relation for gravity waves is satisfied

$$\omega^2 = gk, \tag{2}$$

where g is gravitational acceleration.

The envelope of the wave group is given in the form similar to (1)

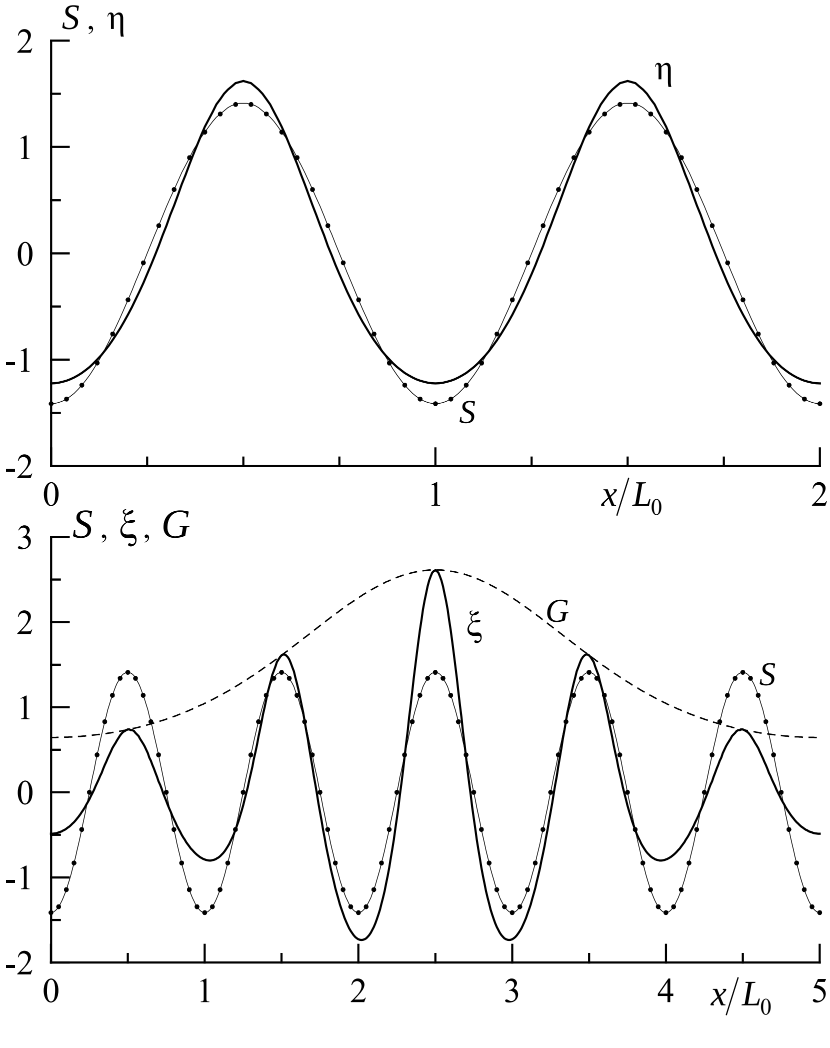
$$G(x,t) = \exp\left[-\mu_1 \cos^2\left(\frac{k_0 x - (\omega_0/2)t}{2\mu_2}\right)\right],$$

where dimensionless parameters μ_1 and μ_2 determine the shape of the envelope of group of waves and set the number of waves in the group. Finally, the profile of the amplitude-modulated asymmetric wave has the following form

$$\xi(x,t) = AG(x,t)\left(\eta(x,t) - \overline{\eta(x,t)}\right),$$

where A is a parameter determining the wave heights; the dash above indicates averaging.

The wave profiles constructed for the harmonic (indicated by index S) and asymmetric waves, as well as for the amplitude-modulated wave, are demonstrated in Fig. 1. To compare them, a normalization is introduced, according to which the dispersions of different types of waves are equal to unity.



F i g. 1. Surface wave profiles constructed depending on the dimensionless distance x/L_0

The distribution of sea surface elevations is nearly Gaussian [21, 22]. When constructing an asymmetric wave, the parameter μ_0 value was chosen so that the skewness

$$\lambda_3 = \overline{\xi^3} \Big/ \overline{\xi^2}^{3/2}$$

was equal to 0.3. The given value λ_3 is close to the upper boundary of the range in which, according to field measurements in different regions of the World Ocean, the values of the skewness of surface elevations lie [23–25].

Wave profile asymmetry effect

If on the sea surface two plane waves of the same frequency *f* propagate towards each other, then a standing wave arises, creating pressure pulsations with 2*f* frequency, which do not decay with depth [5]. The emission of undamped pressure fluctuations by sea surface waves occurs if the condition [10] is satisfied for two waves:

$$\left|\vec{k}_1 + \vec{k}_2\right| \leq \left(\omega_1 + \omega_2\right)/C, \tag{3}$$

where $\vec{k}$ is a wave vector; $\omega = 2\pi f$; indices 1 and 2 denote a compliance with the first and the second waves; *C* is the speed of sound. Since the phase velocity of the surface wave and its harmonics is much less than the speed of sound, condition (3) is satisfied only if the vectors $\vec{k}_1$ and $\vec{k}_2$ are close in absolute value and almost opposite in direction. The wave vector projection of an acoustic wave onto a horizontal plane is $\vec{K} = \vec{k}_1 + \vec{k}_2$. If $k_1 = k_2$ and the angle between the vectors is 180°, then the acoustic wave propagates vertically downward; if equality $k_1 = k_2$ is approximate, then it extends obliquely.

The frequency spectrum of hydroacoustic radiation generated by gravitational waves is described by the expression from [26]

$$S_p\left(\omega_p\right) = \frac{\pi}{2}\left(\frac{\rho g}{C}\right)^2 \omega_p{}^3 S_\xi^2\left(\omega_\xi\right) I,$$

where the frequencies of acoustic ω_p and surface ω_ξ are connected with the relation $\omega_\xi = \frac{1}{2}\omega_p$; ρ is seawater density; $S_\xi\left(\omega_\xi\right)$ is spectrum of surface waves; *I* is an integral determining the level of standing waves. Integral *I* is specified by the expression

$$I = \int \Theta(\varphi)\Theta(\varphi + \pi)\, d\varphi,$$

where $\Theta(\varphi)$ is a wave energy angular distribution function that satisfies the normalization condition $\int \Theta(\varphi)\, d\varphi = 1$; φ is the azimuth angle.

If in the wave field there are two plane sinusoidal waves with a frequency $f = \omega/(2\pi)$, propagating strictly towards each other and having the same amplitudes *A*, then they create an acoustic wave, the amplitude of which is [4]

$$P(2f_0) = 8\pi^2\rho A^2 f_0{}^2. \tag{4}$$

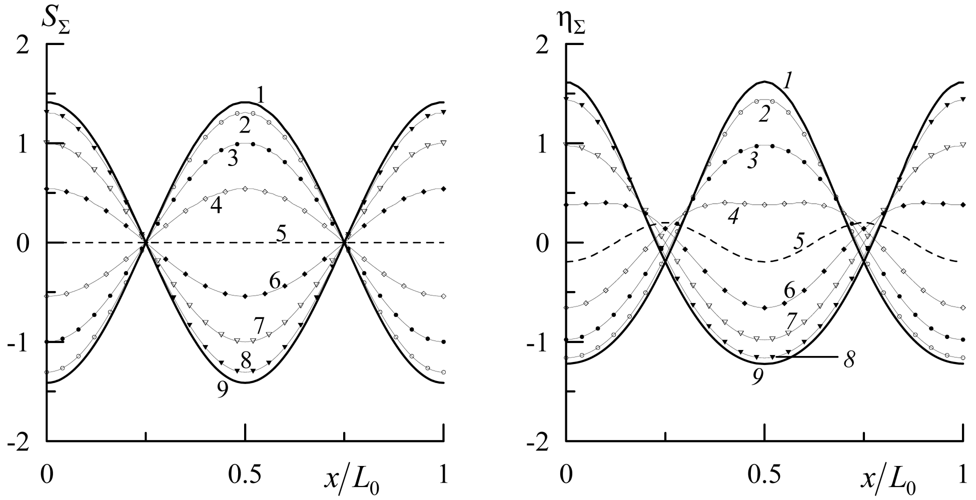
For gravitational waves, expression (4), up to a phase factor, corresponds to expression (6) from [10].

To estimate what effects the skewness of the statistical distribution of surface waves leads to, we consider the situation when a group structure is absent, $G(x,t)\equiv 1$. The profiles of the sea surface created by two symmetric to S_Σ and asymmetric to η_Σ waves propagating towards each other along the *x* axis are given in Fig. 2. They are described by expressions

$$S_\Sigma(x,t)=A_S\left(\cos(\omega_0 t+k_0x)+\cos(\omega_0 t-k_0x)\right),$$

$$\eta_\Sigma(x,t)=A_\eta\left[\exp\left[-\mu_0\cos^2\left(\frac{\omega_0 t+k_0x}{2}\right)\right]+\exp\left[-\mu_0\cos^2\left(\frac{\omega_0 t-k_0x}{2}\right)\right]-2\overline{\eta(x,t)}\right],$$

where the parameters A_S and A_η determine the wave amplitudes; the overline means averaging. Curves 1–9 are plotted with an equal time step δt, which corresponds to a change in the phase of the carrier wave $\omega_0\delta t$ by $\pi/8$.



F i g. 2. Changes in the profile of a sum of two harmonic waves $S_\Sigma(x,t)$ and a sum of two asymmetric waves $\eta_\Sigma(x,t)$. Curves *1–9* correspond to the time moments at which the carrier wave phase changes to $\pi/8$

We expand the wave profile $\eta(x,t=0)$ in a Fourier series. Considering that this function is even, we obtain

$$\eta(x,t=0)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(nk_0x),$$

where a_n are the coefficients of Fourier series; *n* is a number of a harmonic. If the amplitude of the first harmonic is taken as unity, then at $\lambda_3=0.3$ the amplitudes of the second and third spatial harmonics are equal to $a_2=0.14$ and $a_3=0.01$ respectively; at $\lambda_3=0.15$ we get $a_2=0.07$ and $a_3=0.003$.

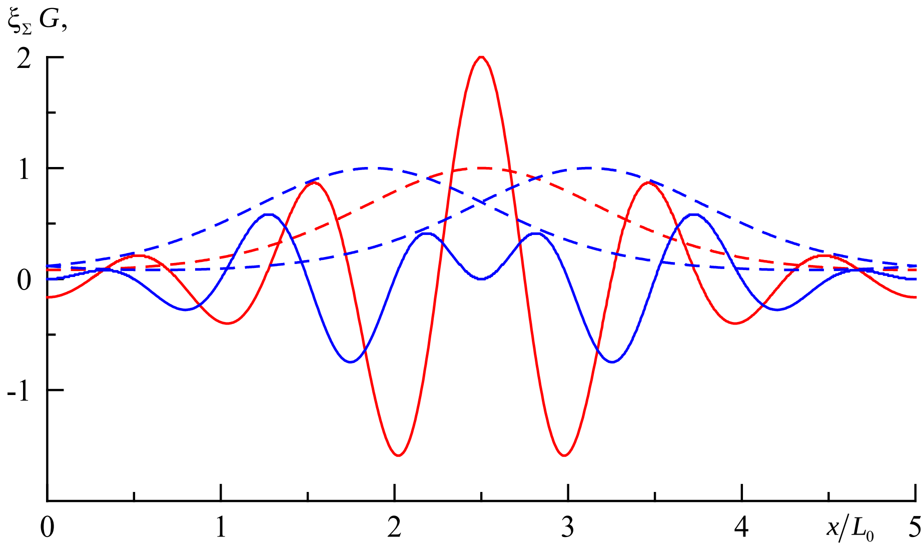
For free waves obeying the dispersion relation (2), the frequencies of the components with wave numbers k_0, $2k_0$, and $3k_0$ are related as $1:\sqrt{2}:\sqrt{3}$. For carrier wave harmonics, the dispersion relation can be written down as

$$\omega_n / k_n = c_0,$$

where c_0 is a phase velocity of a carrier wave; the frequencies of the first three spatial harmonics are related as 1:2:3. The amplitude of the acoustic wave, as follows from (4), is proportional to f^2. As a result, we obtain: at $\lambda_3 = 0.3$ the ratio of acoustic wave amplitudes $P(4f_0)/P(2f_0) = 0.56$, $P(6f_0)/P(2f_0) = 0.012$; at $\lambda_3 = 0.15$ the ratios of amplitudes $P(4f_0)/P(2f_0) = 0.28$, $P(6f_0)/P(2f_0) = 0.003$.

Group structure effect

The fundamental property of the sea surface waves is their group structure, which is expressed in the alternation of large and small waves.



F i g. 3. Wave profiles of a sum of two amplitude-modulated waves – ξ_Σ (solid curves) and G (dashed curves). Red color corresponds to the situation when the envelopes coincide, and blue color – when the envelopes are shifted relative to each other

The group velocity of the waves, which satisfies the dispersion relation (2), is two times less than the phase velocity at which the wave crest moves. The difference between the phase and group velocities leads to the transformation of the wave profile shown in Fig. 3. If at the moment of time $t = 0$ the maxima of the carrier wave and its envelope coincide, then after a time interval $\delta t = 2\pi/\omega_0$ the maximum of the envelope falls on the minimum of the carrier wave, and at $\delta t = 4\pi/\omega_0$ the maxima of the carrier wave and its envelope coincide again.

The main parameter characterizing the group structure of surface waves is the grouping factor *GF*. If the shape of the envelope is known, then the grouping factor can be defined as [27]

$$GF = 1.41\sigma_G/\overline{G} ,$$

where σ_G is a root-mean-square deviation of envelope from is mean value $\overline{G}$. The grouping factor determines the depth of the amplitude modulation of the carrier wave. For the Black Sea, the grouping factor values mainly lie within 0.6–0.9 at an average value of 0.76 [28].

Another parameter that characterizes the group structure of waves is the parameter N_G that determines the number of waves in a group. It can be set as

$$N_G = \overline{\omega}/\overline{\Omega} ,$$

where $\overline{\omega}$ is a mean frequency of wave spectrum; $\overline{\Omega}$ is a mean frequency of envelope spectrum. The values of N_G parameter for the Black Sea mainly lie in the range 4–8 [28].

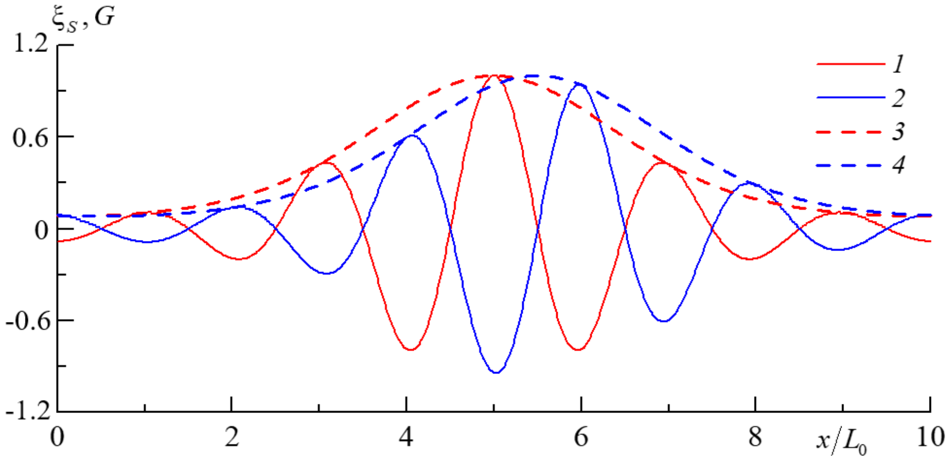
Changes in the surface profile that occur when groups of waves move towards each other are shown in Fig. 3. The profiles are built at $GF = 0.76$ and $N_G = 5$. Two situations are considered when the envelopes of two groups of waves propagating towards each other coincide and when they are displaced relative to each other.

Another factor leading to a nonlinear change in the wave profile is the difference in the phase C_p and group C_g velocities of gravitational waves. In deep water, these velocities, as follows from the dispersion relation (2), are equal to

$$C_p = \frac{\omega}{k} = \sqrt{\frac{g}{k}} , \quad C_g = \frac{d\omega}{dk} = \frac{1}{2}\sqrt{\frac{g}{k}} .$$

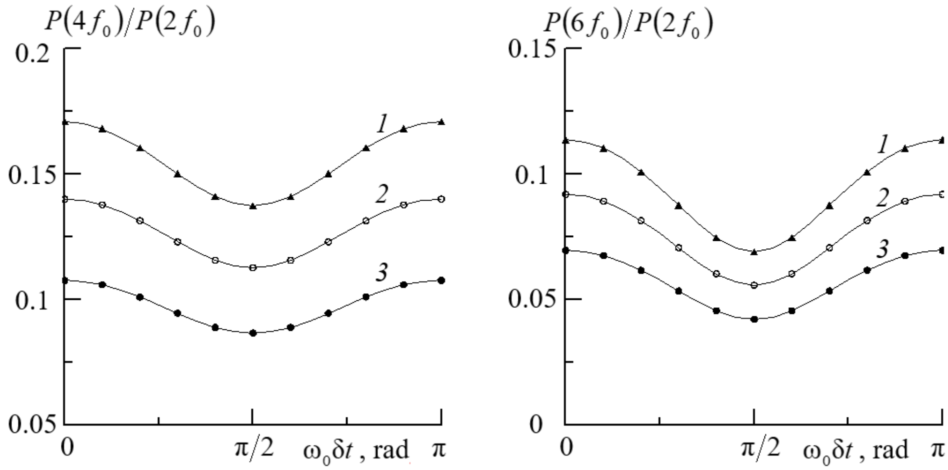
Wave profile variation is demonstrated in Fig. 4.

Wave profile variations lead to a change in the level of the second and third harmonics of the carrier wave. Correspondingly, the ratio of acoustic wave amplitudes changes. To estimate these changes, we expand the wave profile in a Fourier series and, having estimated the amplitudes of the harmonics of surface waves, on the basis of (4) we calculate the ratio of acoustic waves generated by the carrier wave and its second and third harmonics. The relations $P(4f_0)/P(2f_0)$, $P(6f_0)/P(2f_0)$, calculated for the minimum, average and maximum values of the grouping factor, are shown in Fig. 5. Calculations were carried out at $N = 5$.



F i g. 4. Change in the profile of an amplitude-modulated harmonic wave ξ_S. Curves *1* and *2* – ξ_S are constructed with a time shift $\delta t = \pi/\omega_0$; curves *3* and *4* – G are constructed with the same shift δt

For an amplitude-modulated wave, the level of hydroacoustic radiation generated by its second harmonic turned out to be noticeably lower than for an asymmetric wave. As the number of waves increases, the relations $P(4f_0)/P(2f_0)$, $P(6f_0)/P(2f_0)$ decrease.



F i g. 5. Relations between the amplitudes of acoustic waves $P(4f_0)/P(2f_0)$ and $P(6f_0)/P(2f_0)$ generated by the main wave and its harmonics. Curves *1*–*3* correspond to G = 0.6, 0.76 and 0.9

It should be emphasized that the estimates obtained here are preliminary. In particular, they do not take into account intergroup variability, as well as changes in carrier wave frequency within a group. This is due to insufficient knowledge of the spatiotemporal characteristics of surface waves.

Conclusion

The impact of nonlinear effects in the field of wind waves and swell on the generation of acoustic radiation by the sea surface is analyzed. The effects caused by deviations from the model representing the field of sea surface waves as a superposition of linear components are considered. For the analysis, a simplified model, within which it was assumed that the amplitude of the hydroacoustic wave is proportional to the amplitudes of surface waves, was applied.

The skewness of the wave profile (a pointed crest and a flat sole) leads to the appearance of harmonics of the acoustic wave at frequencies higher than the carrier wave frequency. When the value of the skewness of the sea surface elevation distribution is $\lambda_3 = 0.3$ the relation of the amplitudes of acoustic waves $P(4 f_0)/P(2 f_0) = 0.56$, $P(6 f_0)/P(2 f_0) = 0.012$; at $\lambda_3 = 0.15$ the relations of amplitudes are $P(4 f_0)/P(2 f_0) = 0.28$, $P(6 f_0)/P(2 f_0) = 0.003$.

The group structure has a less noticeable effect on the infrasound generation than the asymmetry of the carrier wave. The maximum values of the relations of the amplitudes of acoustic waves generated by the second harmonic and the carrier wave do not exceed the level of 0.17 at the grouping factor $GF = 0.9$.

REFERENCES

1. Duennebier, F.K., Lukas, R., Nosal, E.-M., Aucan, J. and Weller, R.A., 2012. Wind, Waves, and Acoustic Background Levels at Station ALOHA. *Journal of Geophysical Research: Oceans*, 117(C3), C03017. doi:10.1029/2011JC007267

2. Ardhuin, F. and Herbers, T.H.C., 2013. Noise Generation in the Solid Earth, Oceans and Atmosphere, from Nonlinear Interacting Surface Gravity Waves in Finite Depth. *Journal of Fluid Mechanics*, 716, pp. 316-348. doi:10.1017/jfm.2012.548

3. Zapevalov, A.S. and Pokazeev, K.V., 2016. Modeling the Spectrum of Infrasonic Hydroacoustic Radiation Generated by the Sea Surface under Storm Conditions. *Acoustical Physics*, 62(5), pp. 554-558. doi:10.1134/S1063771016050195

4. Salin, B.M. and Salin, M.B., 2019. Calculation of Infrasonic Noise Characteristics by Measuring the Current Values of a Two-Dimensional Wind Wave Field. *Acoustical Physics*, 65(6), pp. 724-730. https://doi.org/10.1134/S1063771019060137

5. Longuet-Higgins, M.S., 1950. A Theory of the Origin of Microseisms. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 243(857), pp. 1-35. doi:10.1098/rsta.1950.0012

6. Potapov, V.A., Tabulevich, V.N. and Chernykh, E.N., 1997. The Impact of Storm Microseismic Vibrations on the Seismicity of the Kuril Islands in the Pacific and Lake Baikal. *Russian Geology and Geophysics*, 38(8), pp. 1411-1419 (in Russian).

7. Ardhuin, F., Stutzmann, E., Schimmel, M. and Mangeney, A., 2011. Ocean Wave Sources of Seismic Noise. *Journal of Geophysical Research: Oceans*, 116(C9), C09004. doi:10.1029/2011jc006952

8. Tabulevich, V.N., Ponomarev, E.A., Sorokin, A.G. and Drennova, N.N., 2001. Standing Sea Waves, Microseisms, and Infrasound. *Izvestiya, Atmospheric and Oceanic Physics*, 37(2), pp. 218-226.

9. Hasselmann, K., 1963. A Statistical Analysis of the Generation of Microseisms. *Review of Geophysics*, 1(2), pp. 177-210. doi:10.1029/RG001i002p00177

10. Brekhovskikh, L.M., 1966. On the Generation of Sound Waves in a Liquid by Surface Waves. *Acoustical Physics*, 12(3), pp. 376-379 (in Russian).

11. Brekhovskikh, L.M., 1966. Sound Waves under Water Caused by Surface Waves in the Ocean. *Izvestiya of the Academy of Sciences of the USSR, Atmospheric and Oceanic Physics*, 2(9), pp. 970-980 (in Russian).

12. Naugolnikh, K.A. and Rybak, S.A., 2003. Sound Generation Due to the Interaction of Surface Waves. *Acoustical Physics*, 49(1), pp. 88-90. doi:10.1134/1.1537393

13. Guralnik, Z., Bourdelais, J., Zabalgogeazcoa, X. and Farrell, W.E., 2013. Wave–Wave Interactions and Deep Ocean Acoustics. *The Journal of the Acoustical Society of America*, 134(4), pp. 3161-3173. doi:10.1121/1.4818782

14. Wilson, J.D., 2018. Modeling Microseism Generation by Inhomogeneous Ocean Surface Waves in Hurricane Bonnie Using the Non-Linear Wave Equation. *Remote Sensing*, 10(10), 1624. doi:10.3390/rs10101624

15. Phillips, O.M., 1961. On the Dynamics of Unsteady Gravity Waves of Finite Amplitude. Part 2. Local Properties of a Random Wave Field. *Journal of Fluid Mechanics*, 11(1), pp. 143-155. doi:10.1017/S0022112061000913

16. Longuet-Higgins, M.S., 1963. The Effect of Non-Linearities on Statistical Distribution in the Theory of Sea Waves. *Journal of Fluid Mechanics*, 17(3), pp. 459-480. doi:10.1017/S0022112063001452

17. Fedele, F. and Tayfun, M.A., 2009. On Nonlinear Wave Groups and Crest Statistics. *Journal of Fluid Mechanics*, 620, pp. 221-239. doi:10.1017/S0022112008004424

18. Gramstad, O. and Trulsen, K., 2007. Influence of Crest and Group Length on the Occurrence of Freak Waves. *Journal of Fluid Mechanics*, 582, pp. 463-472. doi:10.1017/s0022112007006507

19. Yuen, H.C. and Lake, B.M., 1982. Nonlinear Dynamics of Deep-Water Gravity Waves. *Advances in Applied Mechanics*, 22, pp. 67-229. doi:10.1016/s0065-2156(08)70066-8

20. Zapevalov, A.S., 2021. Analytical Representation of a Group Structure Sea Surface Waves. In: T. Chaplina, 2021. *Processes in GeoMedia*–Vol. III. Cham: Springer Geology, pp. 139-145. doi:10.1007/978-3-030-69040-3_14

21. Babanin, A.V. and Polnikov, V.G., 1995. On the Non-Gaussian Nature of Wind Waves. *Physical Oceanography*, 6(3), pp. 241-245. doi:10.1007/BF02197522

22. Zapevalov, A.S., Bol'shakov, A.N. and Smolov, V.E., 2011. Simulating of the Probability Density of Sea Surface Elevations Using the Gram-Charlier Series. *Oceanology*, 51(3), pp. 407-414. doi:10.1134/S0001437011030222

23. Jha, A.K. and Winterstein, S.R., 2000. Nonlinear Random Ocean Waves: Prediction and Comparison with Data. In: ASME, 2000. *Proceedings of the 19th International Offshore Mechanics and Arctic Engineering Symposium*. ASME. Paper No. OMAE 00-6125.

24. Guedes Soares, C., Cherneva, Z. and Antão, E.M., 2004. Steepness and Asymmetry of the Largest Waves in Storm Sea States. *Ocean Engineering*, 31(8-9), pp. 1147-1167. doi:10.1016/j.oceaneng.2003.10.014

25. Zapevalov, A.S. and Garmashov, A.V., 2021. Skewness and Kurtosis of the Surface Wave in the Coastal Zone of the Black Sea. *Physical Oceanography*, 28(4), pp. 414-425. doi:10.22449/1573-160X-2021-4-414-425

About the authors:

Aleksandr S. Zapevalov, Chief Research Associate, Marine Hydrophysical Institute of RAS (2 Kapitanskaya Str., Sevastopol, 299011, Russian Federation,), Dr.Sci. (Phys.-Math.), **Scopus Author ID: 7004433476, Web of Science ResearcherID: V-7880-2017,** sevzepter@mail.ru

The author has read and approved the final manuscript.
The author declares that he has no conflict of interest.