

was equal to 0.3. The given value λ_3 is close to the upper boundary of the range in which, according to field measurements in different regions of the World Ocean, the values of the skewness ratio of surface elevations lie [23–25].

Wave profile asymmetry effect

If on the sea surface two plane waves of the same frequency f propagate towards each other, then a standing wave arises, creating pressure pulsations with $2f$ frequency, which do not decay with depth [5]. The emission of undamped pressure fluctuations by sea surface waves occurs if the condition [10] is satisfied for two waves:

$$|\vec{k}_1 + \vec{k}_2| \leq (\omega_1 + \omega_2)/C, \quad (3)$$

where \vec{k} is a wave vector; $\omega = 2\pi f$; indices 1 and 2 denote a compliance with the first and the second waves; C is the speed of sound. Since the phase velocity of the surface wave and its harmonics is much less than the speed of sound, condition (3) is satisfied only if the vectors \vec{k}_1 and \vec{k}_2 are close in absolute value and almost opposite in direction. The wave vector projection of an acoustic wave onto a horizontal plane is $\vec{K} = \vec{k}_1 + \vec{k}_2$. If $k_1 = k_2$ and the angle between the vectors is 180° , then the acoustic wave propagates vertically downward; if equality $k_1 = k_2$ is approximate, then it extends obliquely.

The frequency spectrum of hydroacoustic radiation generated by gravitational waves is described by the expression from [26]

$$S_p(\omega_p) = \frac{\pi}{2} \left(\frac{\rho g}{C} \right)^2 \omega_p^3 S_\xi^2(\omega_\xi) I,$$

where the frequencies of acoustic ω_p and surface ω_ξ are connected with the relation $\omega_\xi = \frac{1}{2}\omega_p$; ρ is seawater density; $S_\xi(\omega_\xi)$ is spectrum of surface waves; I is an integral determining the level of standing waves. Integral I is specified by the expression

$$I = \int \Theta(\varphi) \Theta(\varphi + \pi) d\varphi,$$

where $\Theta(\varphi)$ is a wave energy angular distribution function that satisfies the normalization condition $\int \Theta(\varphi) d\varphi = 1$; φ is the azimuth angle.

If in the wave field there are two plane sinusoidal waves with a frequency $f = \omega/(2\pi)$, propagating strictly towards each other and having the same amplitudes A , then they create an acoustic wave, the amplitude of which is [4]

$$P(2f_0) = 8\pi^2 \rho A^2 f_0^2. \quad (4)$$

For gravitational waves, expression (4), up to a phase factor, corresponds to expression (6) from [10].

To estimate what effects the skewness of the statistical distribution of surface waves leads to, we consider the situation when a group structure is absent, $G(x,t) \equiv 1$. The profiles of the sea surface created by two symmetric to S_Σ and asymmetric to η_Σ waves propagating towards each other along the x axis are given in Fig. 2. They are described by expressions

$$S_\Sigma(x,t) = A_S (\cos(\omega_0 t + k_0 x) + \cos(\omega_0 t - k_0 x)),$$

$$\eta_\Sigma(x,t) = A_\eta \left[\exp \left[-\mu_0 \cos^2 \left(\frac{\omega_0 t + k_0 x}{2} \right) \right] + \exp \left[-\mu_0 \cos^2 \left(\frac{\omega_0 t - k_0 x}{2} \right) \right] - 2\overline{\eta(x,t)} \right],$$

where the parameters A_S and A_η determine the wave amplitudes; the overline means averaging. Curves 1–9 are plotted with an equal time step δt , which corresponds to a change in the phase of the carrier wave $\omega_0 \delta t$ by $\pi/8$.

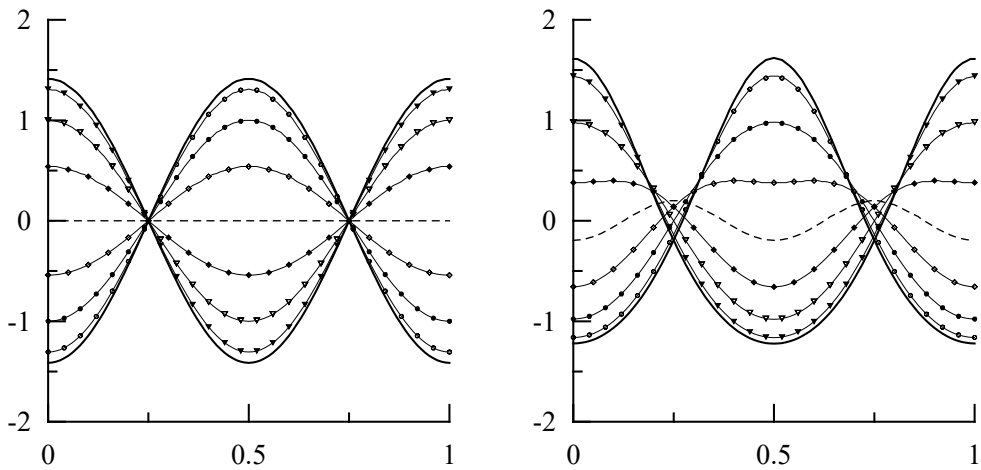


Fig. 2. Changes in the profile of a sum of two harmonic waves $S_\Sigma(x,t)$ and a sum of two asymmetric waves $\eta_\Sigma(x,t)$. Curves 1–9 correspond to the time moments at which the carrier wave phase changes to $\pi/8$

We expand the wave profile $\eta(x,t=0)$ in a Fourier series. Considering that this function is even, we obtain

$$\eta(x,t=0) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nk_0 x),$$

where a_n are the coefficients of Fourier series; n is a number of a harmonic. If the amplitude of the first harmonic is taken as unity, then at $\lambda_3 = 0.3$ the amplitudes of the second and third spatial harmonics are equal to $a_2 = 0.14$ and $a_3 = 0.01$ respectively; at $\lambda_3 = 0.15$ we get $a_2 = 0.07$ and $a_3 = 0.003$.

For free waves obeying the dispersion relation (2), the frequencies of the components with wave numbers k_0 , $2k_0$, and $3k_0$ are related as $1:\sqrt{2}:\sqrt{3}$. For carrier wave harmonics, the dispersion relation can be written down as

$$\omega_n/k_n = c_0,$$

where c_0 is a phase velocity of a carrier wave; the frequencies of the first three spatial harmonics are related as 1:2:3. The amplitude of the acoustic wave, as follows from (4), is proportional to f^2 . As a result, we obtain: at $\lambda_3 = 0.3$ the ratio of acoustic wave amplitudes $P(4f_0)/P(2f_0) = 0.56$, $P(6f_0)/P(2f_0) = 0.012$; at $\lambda_3 = 0.15$ the ratios of amplitudes $P(4f_0)/P(2f_0) = 0.28$, $P(6f_0)/P(2f_0) = 0.003$

Group structure effect

The fundamental property of the sea surface waves is their group structure, which is expressed in the alternation of large and small waves.

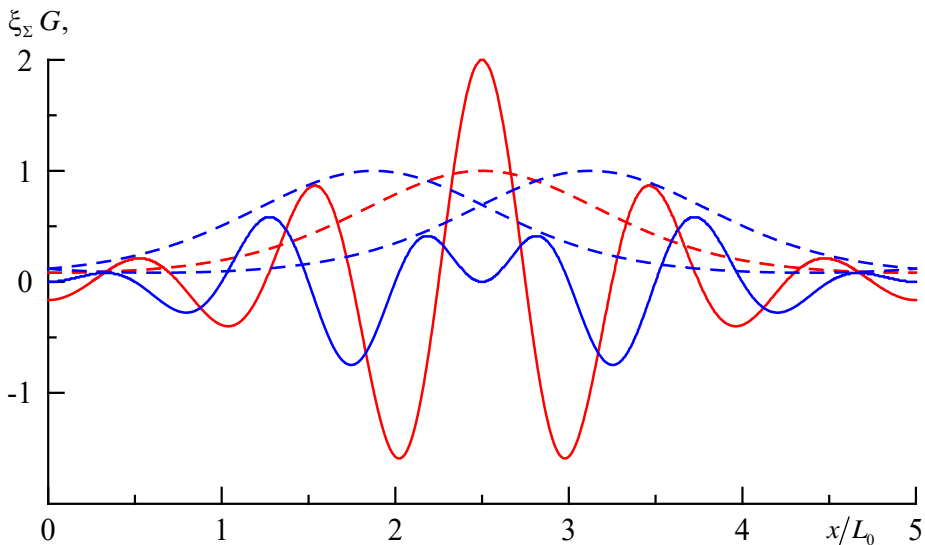


Fig. 3. Wave profiles of a sum of two amplitude-modulated waves – ξ_{Σ} (solid curves) and G (dashed curves). Red color corresponds to the situation when the envelopes coincide, and blue color – when the envelopes are shifted relative to each other

The group velocity of the waves, which satisfies the dispersion relation (2), is two times less than the phase velocity at which the wave crest moves. The difference between the phase and group velocities leads to the transformation of the wave profile shown in Fig. 3. If at the moment of time $t = 0$ the maxima of the carrier wave and its envelope coincide, then after a time interval $\delta t = 2\pi/\omega_0$ the maximum

of the envelope falls on the minimum of the carrier wave, and at $\delta t = 4\pi/\omega_0$ the maxima of the carrier wave and its envelope coincide again.

The main parameter characterizing the group structure of surface waves is the grouping factor GF . If the shape of the envelope is known, then the grouping factor can be defined as [27]

$$GF = 1.41\sigma_G/\bar{G},$$

where σ_G is a root-mean-square deviation of envelope from its mean value \bar{G} . The grouping factor determines the depth of the amplitude modulation of the carrier wave. For the Black Sea, the grouping factor values mainly lie within 0.6–0.9 at an average value of 0.76 [28].

Another parameter that characterizes the group structure of waves is the parameter N_G that determines the number of waves in a group. It can be set as

$$N_G = \bar{\omega}/\bar{\Omega},$$

where $\bar{\omega}$ is a mean frequency of wave spectrum; $\bar{\Omega}$ is a mean frequency of envelope spectrum. The values of N_G parameter for the Black Sea mainly lie in the range 4–8 [28].

Changes in the surface profile that occur when groups of waves move towards each other are shown in Fig. 3. The profiles are built at $GF = 0.76$ and $N_G = 5$. Two situations are considered when the envelopes of two groups of waves propagating towards each other coincide and when they are displaced relative to each other.

Another factor leading to a nonlinear change in the wave profile is the difference in the phase C_p and group C_g velocities of gravitational waves. In deep water, these velocities, as follows from the dispersion relation (2), are equal to

$$C_p = \frac{\omega}{k} = \sqrt{\frac{g}{k}}, \quad C_g = \frac{d\omega}{dk} = \frac{1}{2}\sqrt{\frac{g}{k}}.$$

Wave profile variation is demonstrated in Fig. 4.

Wave profile variations lead to a change in the level of the second and third harmonics of the carrier wave. Correspondingly, the ratio of acoustic wave amplitudes changes. To estimate these changes, we expand the wave profile in a Fourier series and, having estimated the amplitudes of the harmonics of surface waves, on the basis of (4) we calculate the ratio of acoustic waves generated by the carrier wave and its second and third harmonics. The relations $P(4f_0)/P(2f_0)$, $P(6f_0)/P(2f_0)$, calculated for the minimum, average and maximum values of the grouping factor, are shown in Fig. 5. Calculations were carried out at $N = 5$.

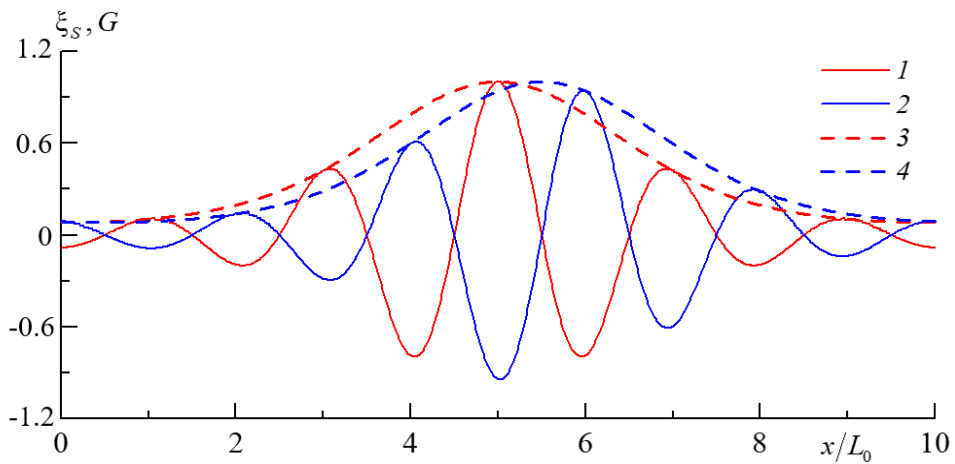


Fig. 4. Change in the profile of an amplitude-modulated harmonic wave ξ_S . Curves 1 and 2 – ξ_S are constructed with a time shift $\delta t = \pi/\omega_0$; curves 3 and 4 – G are constructed with the same shift δt

For an amplitude-modulated wave, the level of hydroacoustic radiation generated by its second harmonic turned out to be noticeably lower than for an asymmetric wave. As the number of waves increases, the relations $P(4f_0)/P(2f_0)$, $P(6f_0)/P(2f_0)$ decrease.

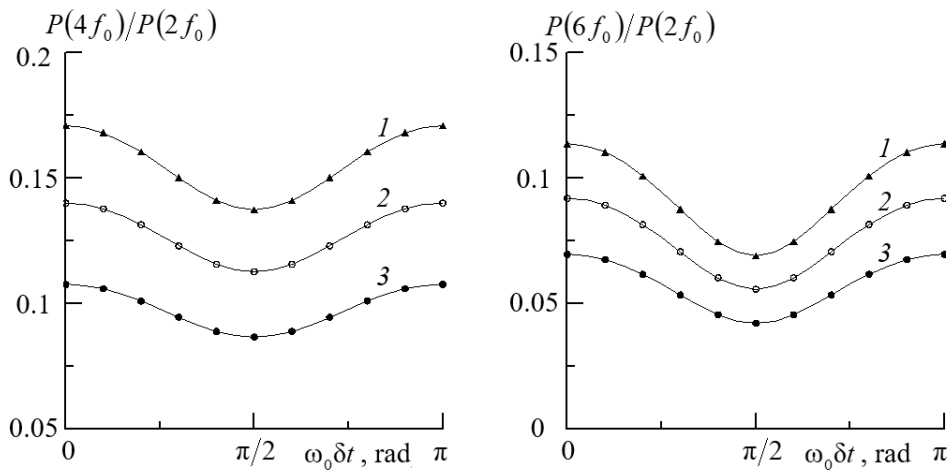


Fig. 5. Relations between the amplitudes of acoustic waves $P(4f_0)/P(2f_0)$ and $P(6f_0)/P(2f_0)$ generated by the main wave and its harmonics. Curves 1–3 correspond to $G = 0.6, 0.76$ and 0.9

It should be emphasized that the estimates obtained here are preliminary. In particular, they do not take into account intergroup variability, as well as changes in carrier wave frequency within a group. This is due to insufficient knowledge of the spatiotemporal characteristics of surface waves.

Conclusion

The impact of nonlinear effects in the field of wind waves and swell on the generation of acoustic radiation by the sea surface is analyzed. The effects caused by deviations from the model representing the field of sea surface waves as a superposition of linear components are considered. For the analysis, a simplified model, within which it was assumed that the amplitude of the hydroacoustic wave is proportional to the amplitudes of surface waves, was applied.

The skewness of the wave profile (a pointed crest and a flat sole) leads to the appearance of harmonics of the acoustic wave at frequencies higher than the carrier wave frequency. When the value of the skewness ratio of the sea surface elevation distribution is $\lambda_3 = 0.3$ the relation of the amplitudes of acoustic waves $P(4f_0)/P(2f_0) = 0.56$, $P(6f_0)/P(2f_0) = 0.012$; at $\lambda_3 = 0.15$ the relations of amplitudes are $P(4f_0)/P(2f_0) = 0.28$, $P(6f_0)/P(2f_0) = 0.003$.

The group structure has a less noticeable effect on the infrasound generation than the asymmetry of the carrier wave. The maximum values of the relations of the amplitudes of acoustic waves generated by the second harmonic and the carrier wave do not exceed the level of 0.17 at the grouping factor $GF = 0.9$.

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