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Original article

Excitation of Internal Waves in a Shallow Sea Basin with an Open Inlet under Conditions of Parametric Resonance

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Abstract
Purpose. The purpose of the study is to show (using the *in situ* measurement data) that in autumn under parametric resonance conditions, internal waves (IW) are excited in Posyet Bay under the influence of seiche vibrations of its level.
Methods and Results. The spectral analysis methods have revealed a number of IW frequencies close to those of the most intensive seiche vibrations of the bay. The Mathieu equation was obtained and analyzed for the horizontal component of IW orbital velocity. For the conditions for observing IW, the necessary and sufficient conditions for implementing the parametric resonance in the model basin approximating Posyet Bay, were formulated. Verification of these conditions has shown that in autumn both necessary and sufficient conditions of the parametric resonance between the IW and sea level seiche vibrations are fulfilled in the bay.
Conclusions. The experimental data indicate that in the autumn season a number of IW frequencies are close to those of free oscillations of the sea level in Posyet Bay. It is shown that the barotropic currents induced by seiche vibrations can excite internal waves by means of parametric resonance.

Keywords: seiches, barotropic wave current, Posyet Bay, internal waves, parametric instability, parametric resonance

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Introduction

Internal waves (IW) play a significant role in the mixing processes occurring in the surface and bottom boundary layers, participating in their formation [1, 2]. Along with winter convection, these waves play an essential role in the processes of heat and mass transfer in the surface layer of ice-covered basins [3].

As is known [1] [4], excitation of IW with frequencies of ~ 0.7 N_{max}, where N_{max} is the maximum value of buoyancy frequency in the basin, occurs due to pressure pulsations or tangential wind stress. This paper presents an alternative mechanism for the excitation of such waves. It is based on the phenomenon of IW parametric

[1] Miropol'sky, Yu.Z., 1981. *Dynamics of Internal Gravity Waves in the Ocean*. Leningrad: Gidrometeoizdat, 302 p. (in Russian).

instability, which can be caused by external effects such as seiche vibrations of the basin free surface. This mechanism is particularly effective under conditions of parametric resonance, which is a special type of parametrically excited oscillations.

The study of IW parametric instability in a stratified fluid is a relatively recent development, having only commenced in the last few decades [5]. The cited work presents a number of considerations regarding the potential for high-frequency disturbances to increase in the presence of a low-frequency internal wave. A theoretical study of the parametric instability of a weakly nonlinear internal wave is presented in [6]. The present study demonstrates that an internal wave of finite amplitude can be unstable. In the studies [7, 8], the use of *in situ* data enabled the determination that the steepening of the leading edge of a semidiurnal internal wave in Posyet Bay coastal zone results in the effective generation of its harmonics with periods $T_n = 12.4/n$ (h), $n = 2, 3, 4, \ldots$.

In the present work, the necessary and sufficient conditions for IW excitation by means of parametric resonance are obtained analytically for the case of long internal and surface waves in a sea basin with a semi-open boundary. It is demonstrated that the physical nature of this excitation mechanism is constituted by the parametric amplification of the IW amplitude due to modulation of its horizontal orbital velocity component, which is caused by the barotropic current induced by seiche vibrations. This method of wave generation in a stably stratified fluid differs significantly from the widely known ones [9, 10] and is implemented without introducing additional anisotropy into the system. This ensures, in particular, the absence of spatial dissipation of energy transferred by the IW. *In situ* data are employed to analyse the feasibility of implementing the necessary and sufficient conditions for parametric resonance between the field of internal waves and the barotropic wave current generated by the Helmholtz mode and subsequent modes of seiche vibrations in Posyet Bay.

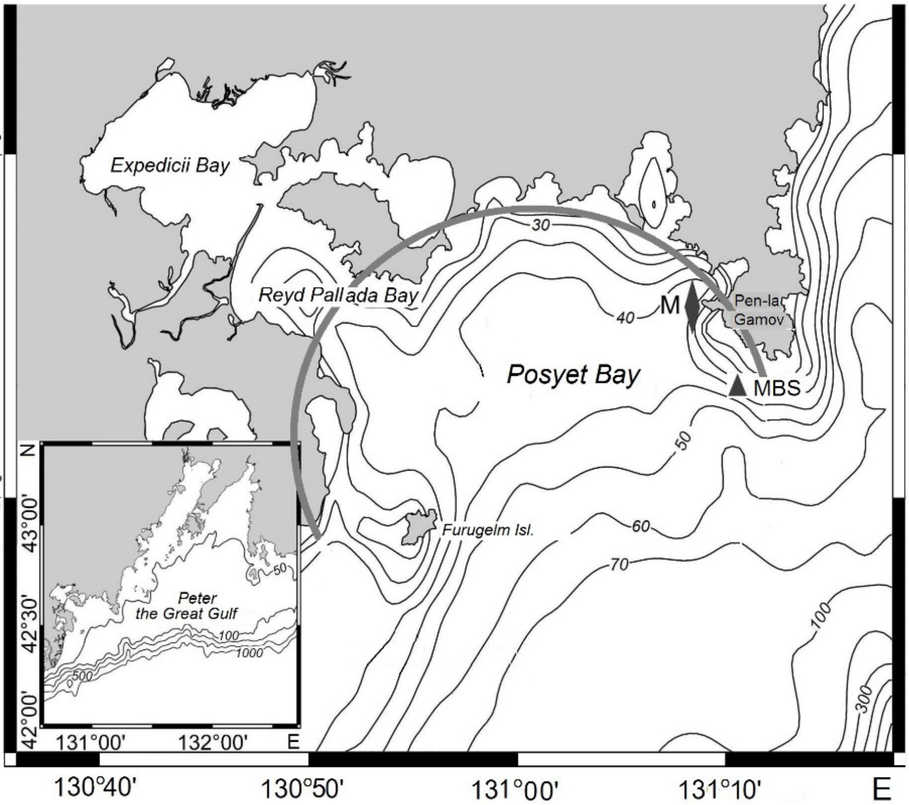
The objective of this study is to examine the process of internal wave parametric excitation in a shallow sea basin, specifically the role of seiche vibrations in hydrodynamic systems under conditions of parametric resonance. The study draws upon theoretical concepts of parametric resonance and field observations carried out in Posyet Bay over several years.

Study area and measurement data

The frequency content of seiche vibrations was analyzed using the data obtained from a tide gauge. The measurement error was 0.5 cm and the sampling interval was 7.5 min in October 2001 and 1 min in August 2003. The tide gauge was installed in the coastal zone of the Gamov Peninsula, within the Posyet Bay area. Its position is indicated by a diamond-shaped symbol on the map of the bay (Fig. 1). The map also provides the bathymetry of the bay obtained from navigational charts of the bay and its adjacent areas [2]. The area of water adjacent to the bay is limited by a semicircle

[2] Russian Emergencies Ministry, 2009. [*Atlas of the Peter the Great Gulf and Northwestern Coast of the Sea of Japan to the Sokolovskaya Bay (for Small-Size Vessels)*]. Vladivostok: GIROSKOP, 61 p. (in Russian).

with a radius of L ~ 13.5 km. As indicated on the navigational chart, the depth of the bay at its entrance is ~ 45–50 m.

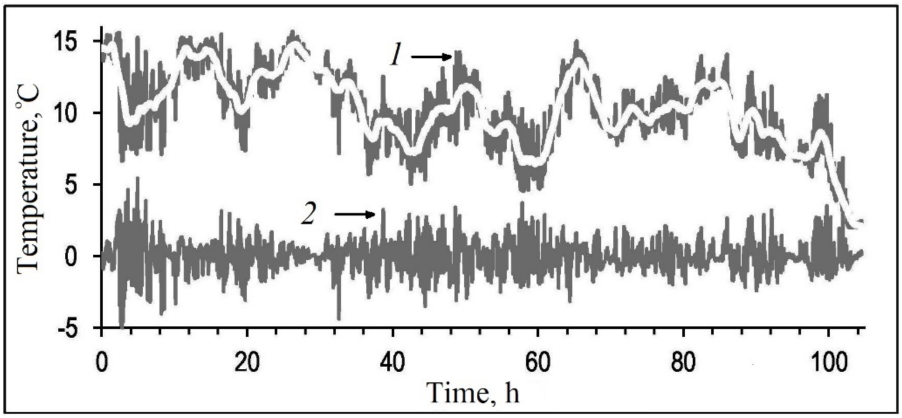


F i g. 1. Map-diagram of Posyet Bay. The inset shows Peter the Great Gulf

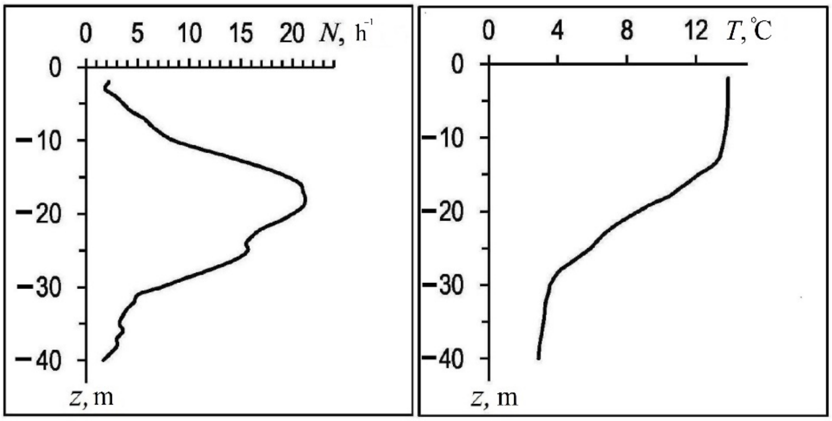
The study of internal waves was performed according to the measured data of the vertical section of the temperature field using a moored buoy station (MBS) deployed on the 40 m isobath on 14 September 2013. The geographical location of the MBS is indicated by a black triangle in Fig. 1. It was equipped with nine HOBO thermographs spaced 4 m apart from the surface. The HOBO autonomous digital thermograph, manufactured by *Onset*, has an accuracy of 0.21°C in the range 0–50°C and a resolution of 0.02°C at 25°C, as well as 64 kB of memory (~ 42,000 12-bit temperature measurements). Temperature recording at the stations was carried out with a resolution of 1 min. Duration of the measurements was just over 10 days.

Fig. 2 shows a 5-day temperature realization at z = – 24 m horizon, recorded by the MBS thermograph, and its low-frequency trend. The realization of high-frequency temperature variations is also shown.

In the area of the buoy stations, 8 hydrological probings with a discreteness of 3 h were performed on 13 September 2013. The probings were performed with a Canadian RBR XRX-620 probe.



F i g. 2. Temperature near the moored buoy station (*1*), its low-frequency trend (white color graph) and high-frequency pulsations (*2*)



F i g. 3. Mean daily profiles of buoyancy frequency *N* (left) and temperature (*T*) (right) in the vicinity of buoy station

Fig. 3 (left) shows the typical buoyancy frequency profile for the autumn season in Posyet Bay. The presented profile *N*(*z*) was used to calculate the phase velocity of the lowest IW mode with the frequencies of the bay seiche oscillations. The analysis of the daily mean temperature profile (Fig. 3, right) showed that the background conditions at the horizon z_1 = –24 m during the experiment near the MBS were characterised by a quasi-linear temperature dependence with depth.

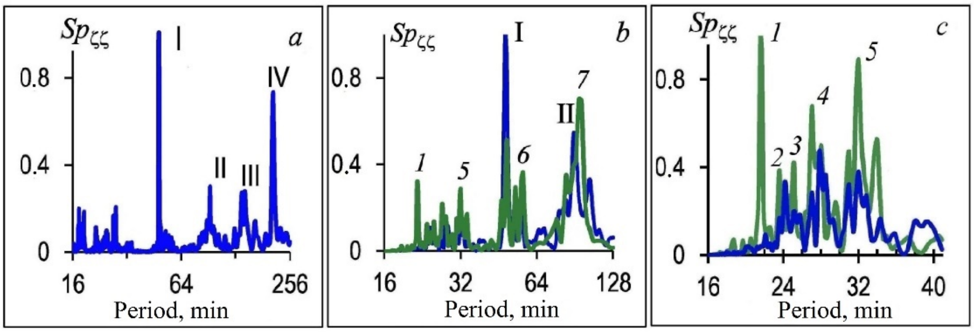
Methodology of spectral data processing and its results

The characteristic time scales of sea level seiche oscillations (ζ) and temperature pulsations (*T*, °C) in the bay caused by internal waves were determined using standard spectral analysis methods [3] [11]. The ζ and *T* fluctuations were separated

[3] Dragan, Ya.P., Rozhkov, V.A. and Yavorskyi, I.M., 1987. *Methods of Probabilistic Analysis of Oceanological Process Rhythms*. Leningrad: Gidrometeoizdat, 319 p. (in Russian).

into a high-frequency component and a low-frequency trend by a Hamming filter with a window of 256 min duration. The low-frequency trend realizations obtained after filtering served as the background for determining the frequencies of internal waves and seiche oscillations with periods of 8–256 min. The realizations with the frequencies of the seiche oscillations were calculated as the difference between the initial level and temperature realizations and those of the low-frequency trend ζ and *T*. The resulting time series of the ζ and *T* fluctuations were used to calculate the spectral densities (hereafter referred to as spectra) of the level fluctuations ($Sp_{\zeta\zeta}$) and temperature pulsations ($Sp_{\gamma\gamma}$).

The spectra of the bay level fluctuations are normalized to the maximum value falling on the period of ~ 47 min (Fig. 4, *a*, *b*) and ~ 22 min (Fig. 4, *c*). The spectrum with the maximum at period T_0~ 47 min and the spectrum with a less pronounced broadband maximum at period T_1 ~ 93 min are shown in blue. The spectrum with the dominant maximum located at period T_7 ~ 96 min (Fig. 4, *b*) and the dominant maximum at period T_1 ~ 22 min (Fig. 4, *c*) are highlighted in green. The spectrum in Fig. 4, a is calculated from a two-week realization, in Fig. 4, *b*, *c* – from two consecutive weekly realizations obtained in October 2001.



F i g. 4. Normalized spectra of the Posyet Bay level fluctuations in August 2003 (*a*) and October 2001 (*b*, *c*)

The spectrum in Fig. 4, a is characterized by a delta-shaped maximum at the period T_0 ~ 47 min, marked with the Roman numeral I, and a less intense broadband maximum at the period T_1 ~ 96 min marked with the Roman numeral II. We recorded two maxima in the range of periods exceeding 100 min; they are marked with the Roman numerals III and IV.

Let us now consider the spectra shown in Fig. 4, *b*, *c*, obtained in 2001 for two consecutive 7-day realizations. In the range of 16–28 min periods, the corresponding maxima are numbered *1*, *2*, …, *7*. Here we give the values of the periods in which these maxima are located:

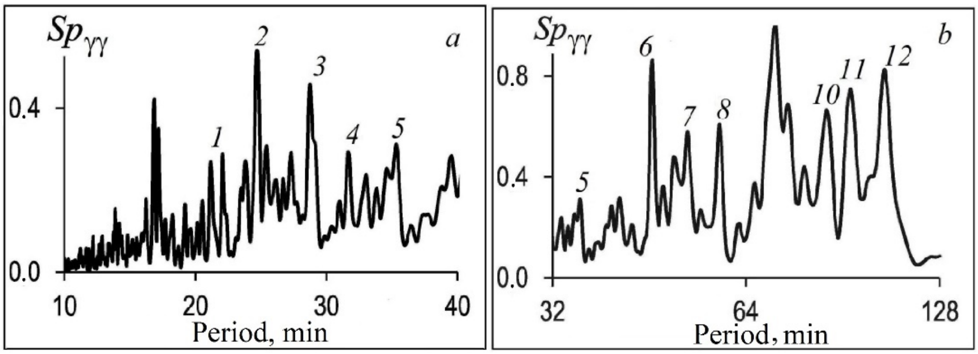
Spectrum maxima	*1*	*2*	*3*	*4*	*5*	*6*	*7*
T_m, min	22	24	25	27	32	47	96

Thus, as a result of the spectral analysis, intense manifestations of water level fluctuations in the bay were identified at frequencies ν_0 ~ 47 min^{-1} and

ν_0^+ ~ 1/96 min^{-1}, as well as less intense manifestation at frequencies ν_1 ~ 1/32 min^{-1}, ν_2 ~ 1/27 min^{-1} and ν_3 ~ 1/25 min^{-1}.

We will consider the spectral composition of temperature pulsations in the bay. We will present the results of calculating the energy spectrum of these pulsations in the ranges of 10–40 and 32–128 min^{-1}, i.e. in the same ranges as the fluctuations of their level. The spectral analysis was carried out according to the implementation of high-frequency temperature pulsations recorded by the MBS thermograph at the horizon z_1= –20 m (Fig. 2).

Fig. 5 shows the spectrum normalized to the maximum value of the temperature pulsations recorded by the MBS at the z = –24 m horizon. The numbers *1–12* indicate the numbers of the corresponding spectral maxima on a low-frequency background, showing the modulation of these pulsations by the low-frequency component.



F i g. 5. Temperature pulsation normalized spectrum in the ranges 10–40 min (*a*) and 32–128 min (*b*)

Here are the values of T_m (min) periods of $Sp_{\gamma\gamma}$ spectral maxima shown in Fig. 5:

Spectral maxima	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
T_m, min	22	25	28	32	35	46	52	58	71	85	93	105

In the spectra, the most noticeable feature is the narrow-band maximum at the frequency ν_0 ~1/46 min^{-1} with m = 6. It should also be noted that the maxima at the frequencies ν_1 ~ 1/25 min^{-1} and ν_2 ~1/28 min^{-1} are close to those at the frequencies of the bay seiche oscillations ~ 1/25 min^{-1} and ~ 1/27 min^{-1}.

If we analyze the temperature pulsation spectrum shown in Fig. 3, *a*, we notice that the differences between the frequencies $\nu_1 - \nu_0$ and $\nu_2 - \nu_0$ are close to the maximum frequencies in the spectrum at 52 and 71 min^{-1} periods. In other words, for the frequencies corresponding to these periods the following approximate relationships are fulfilled: $\nu_1 - \nu_0$ ~ 1/55 min^{-1} and $\nu_2 - \nu_0$ ~ 1/72 min^{-1}. It should also be noted that the frequencies ν_0 ~1/46 min^{-1}, ν_1^- ~ 1/105 min^{-1}, ν_2^- ~ 1/180 min^{-1}, around which the spectral maxima are located, satisfy the approximate expressions $\nu_n \sim \nu_0 + \nu^-_n$ where n is equal to 1 and 2, ν_1 ~1/32 min^{-1} and ν_2 ~1/37 min^{-1}.

The observed features of the spectra in the area of the buoy station deployment may be an indirect indication of the IW parametric instability, caused, among other things, by seiche oscillations.

Parametric excitation of internal waves in a shallow sea basin by seiche oscillations of its free surface

We introduce a rectangular coordinate system with *z*-axis directed vertically upwards; *x*-axis is compatible with the velocity direction of the barotropic one-dimensional current of stratified fluid. The system of hydrodynamic equations for sufficiently long linear IWs in the Boussinesq approximation in the specified flow has the following form [4, 10]:

$$D_0 u = \frac{1}{\rho_0}\frac{\partial p}{\partial x}, \qquad \frac{1}{\rho_0}\frac{\partial p}{\partial z} - b = 0, \tag{1}$$

$$D_0 \rho = w\frac{d\rho_0}{dz}, \qquad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0. \tag{2}$$

Here $D_0 = \partial/\partial t + U\,\partial/\partial x$, U is barotropic current velocity; u and w are horizontal and vertical components of IW orbital velocity; p and ρ are wave disturbances of pressure and density; $\rho_0(z)$ is average density of liquid layer; $b = \rho g/\rho_0$ is wave fluctuations of buoyancy per unit volume. We transform the system of equations (1), (2) into a single equation for u of the following form:

$$D_0^2\left(\frac{\partial^2 u}{\partial z^2} - \frac{2}{N}\frac{dN}{dz}\frac{\partial u}{\partial z}\right) + N^2\frac{\partial^2 u}{\partial x^2} = 0, \tag{3}$$

where $N(z) = \left(g\, d\ln\rho_0/dz\right)^{1/2}$ is buoyancy frequency.

Since the system of equations (1), (2) is horizontally homogeneous, the solution of equation (3) is described by a superposition of IW modes of arbitrary shape $u_m \sim \psi(c_m t)\varphi_m(z)\exp(ikx)$. In this expression $\psi_m(t)$ is the amplitude function of the wave mode with the number m; $\varphi_m(z)$ and c_m are the eigenfunction and eigenvalue of the boundary value problem

$$\frac{d^2\varphi_m}{dz^2} - \frac{2}{N}\frac{dN}{dz}\frac{d\varphi_m}{dz} + \frac{N^2}{c_m^2}\varphi_m = 0, \qquad \varphi_m(0) = \varphi_m(-H) = 0. \tag{4}$$

Here it is assumed that the bottom ($z = -H$) and the free surface ($z = 0$) are rigid walls.

For the function $\psi_m(t)$ (hereinafter we omit the index m, assuming $m = 1$), taking into account the orthogonality of the set of functions $\varphi_m(z)$, after a series of transformations we obtain the equation

$$\frac{d^2\psi}{dt^2} + 2i(kU)\frac{d\psi}{dt} + \left[(kU)^2 + (kc_{ph})^2\right]\psi = 0, \tag{5}$$

which we reduce to normal form using the transformation $\psi(t) = \zeta(t)\exp\left(i\int kUdt\right)$. As a result, we obtain the following equation for the function $\zeta(t)$:

$$d^2\zeta/dt^2 + \left[\left(kc_{ph}\right)^2 - i\left(k\,dU/dt\right)^2\right]\zeta = 0\,. \qquad (6)$$

Next, we define the velocity of a barotropic current pulsating with a frequency ω as follows: $U = u_0\exp(i\omega t)$. Then the imaginary term in the square brackets of equation (6) is equal to ωkU. We represent the equation (6) solution as the sum of real and imaginary parts. In this case, the real part of the solution (denoted as $\eta = \mathrm{Re}(\zeta)$) satisfies the equation

$$d^2\eta/dt^2 + \Omega_0^2\left(1 + \mu\sin(\Omega t)\right)\eta = 0\,, \qquad (7)$$

where $\mu = (u_0/c_{ph})(\Omega/\Omega_0)$, and dimensional quantities (indicated by dashes) have the form $\eta = \eta'/H$, $t = t'/N_{\max}$, $\Omega = \omega/N_{\max}$, $\Omega_0 = \omega_0/N_{\max}$, $\omega_0 = kc_{ph}$ is internal wave frequency.

Thus, when an internal wave of a fixed (lowest) mode propagates in a barotropic current pulsating with a frequency ω, the real part of its amplitude function evolves according to equation (7).

Equation (7) is the well-known Mathieu equation. Its general solution has the following form [4]

$$\eta(t) = C_1\exp(-i\sigma t)\Phi(t) + C_2\exp(-i\sigma t)\Phi(-t)\,, \qquad (8)$$

where C_1, C_2 are constants; $\Phi(t)$ and $\Phi(-t)$ are periodic functions. The value σ characterizes the growth rate of the solution (8) and is a complex function of ω_0 and μ parameters. In this case, the solution (8) grows exponentially with time. The phenomenon consisting in the increase in oscillations of hydrodynamic system parameters is called parametric resonance.

Next, we show that in a sea basin affected by weak periodic fluctuations in the velocity of barotropic current $U = u_0\sin(\omega t)$ induced by seiche level vibrations, the IW parametric generation with c_{ph} phase velocity is possible under condition $u_0^2 \ll c^2_{ph}$. During the generation process, the wave amplitude, specified by the function $\eta(t)$, is described by equation (7). We will seek a solution to this equation in the region of the main demultiplication resonance, i.e. when condition $\left|\Omega_0 - \Omega/2\right| \le \mu$ is satisfied in the following form

$$\eta(t) = A(t)\sin\left[\Omega t/2 - \theta(t)\right]. \qquad (9)$$

Using the Krylov – Bogolyubov averaging method [5], for the amplitude A and phase θ we obtain a system of equations

[4] Yakubovich, V.A. and Starzhinskii, V.M., 1987. [*Parametric Resonance in Linear Systems*]. Moscow: Nauka, 328 p. (in Russian).

[5] Krylov, N.M. and Bogolyubov, N.N., 1937. *Introduction to Non-Linear Mechanics*. Kyiv: Publishing House of the Academy of Sciences of the Ukrainian SSR, 363 p. (in Russian).

$$dA/dt = \varepsilon A \cos(2\theta), \qquad d\theta/dt = \delta - \varepsilon \cos(2\theta), \tag{10}$$

where $\varepsilon = -\mu\Omega_0/4$; $\delta = \Omega_0 - \Omega/2$. System (8) has the following invariant

$$I = A^2 (d\theta/dt) = \text{const}, \tag{11}$$

which allows it to be easily integrated. It turns out that when the condition $\varepsilon^2 > \delta^2$ is satisfied, a solution of the form $A \sim \exp\left(t\sqrt{\varepsilon^2 - \delta^2}\right)$ exists. This can be verified by simply substituting the indicated solution into equation (7). Thus, the amplitude of the IW fixed mode is proportional to $\exp\left(t\sqrt{\varepsilon^2 - \delta^2}\right)$, and the condition for its small-amplitude exponential growth of horizontal velocity of wave currents is the condition $|\delta| < \varepsilon$, which corresponds to the parametric instability criterion of a pendulum oscillations with a vibrating suspension point in the absence of friction [6]. In addition, the smallness condition of parameter $\mu \ll 1$ and parametric resonance of frequencies $|\delta| < \min\{\mu, \varepsilon\}$ must be met. Hence, taking into account the inequality $\mu \ll 1$, the condition of IW amplitudes "swinging" by a depth-uniform pulsating flow with a frequency Ω and a maximum value of its velocity u_0 takes the form

$$|\Omega_0 - \Omega/2| = \Omega u_0 / 4c_{ph}\ . \tag{12}$$

We can easily see that parametric resonance should take place at any $\omega = n\omega_0/2$ (where n is an integer), including $n = 2$. In this case, the boundaries of the parametric generation second zone are determined by the inequalities from the work [7]:

$$-5\mu^2\omega/24 < \omega_0 - \omega < \mu^2\omega/4, \tag{13}$$

where ω is a frequency of pulsating barotropic current.

In conclusion, we formulate the necessary and sufficient conditions under which IW parametric generation is realized in a shallow basin being affected by the modulation of its horizontal component of the orbital motion velocity caused by the seiche vibrations (SV) barotropic current of the basin free surface.

Parametric generation of fixed-mode IWs with phase velocity c_{ph} and wave number k in a sea basin of depth H by a field of standing surface waves with frequency ω is possible if the following conditions are met:

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[6] Landau, L.D. and Lifshitz, E.M., 1969. *Mechanics*. Oxford, London, Edinburgh, New York: Pergamon Press, 165 p.

[7] Rabinovich, M. I. and Trubetskov, D.I., 1984. *Introduction to the Theory of Oscillations and Waves*. Moscow: Nauka, 432 p. (in Russian).

– the lengths of internal (λ_{int}) and surface (λ_{sur}) waves significantly exceed the basin depth H, i.e. $H << \lambda_{int} << \lambda_{sur}$, and the IWs frequency range is limited by the frequency $\omega_*/2$, where ω_* is the lowest frequency of basin seiche oscillations;

– the IW phase velocity (c_{ph}) significantly exceeds the maximum value of barotropic current velocity (u_0), i.e. their ratio $\mu = (u_0/c_{ph}) << 1$. The mismatch between the IW frequency $\omega_0 = kc_{ph}$ and that of seiche oscillations ω should not exceed the product $\mu\omega$, i.e. $|\omega_0 - \omega| < \mu\omega$.

Using *in situ* data, we will demonstrate the formation of necessary and sufficient conditions for the IW excitation under effect of parametric resonance caused by the fundamental zero mode (Helmholtz mode) (as well as the first, second and subsequent SV modes of the bay water mass) in a model basin with a semicircular water area approximating Posyet Bay in autumn.

Discussion

Spectral analysis of temperature pulsations caused by the IW field in the bay showed that a number of frequencies of these pulsations are close to those of seiche level oscillations. Consequently, the necessary condition for IW parametric excitation by seiche oscillations is fulfilled.

In the autumn period, which is characterized by intense seiche oscillations, a sufficient condition for parametric resonance implementation between wave movements is fulfilled. Consequently, the IWs are excited in Posyet Bay in this period under SV effect.

Let us turn now to the data of *in situ* level measurements in the bay. Fig. 4 shows the spectrum of free surface fluctuations of the bay in the 1/16–1/256 min^{-1} frequency range, which is typical for October. Two dominant maxima at periods of 47 and 92 min and three less pronounced maxima at periods of ~ 33, ~ 28 and ~ 25 min, respectively, stand out in the spectrum. It should be noted that the ratio of these periods to T_0 ~ 47 min period is ~ 0.7, ~ 0.6 and ~ 0.5.

A number of experimental studies [12–15] revealed that the Helmholtz mode, a longitudinal fluctuation of barotropic current level and velocity with T_0 period, directed along the normal to the open boundary, has the greatest intensity in a basin with a semi-enclosed water area. For the basins of the simplest form, the periods of the first and subsequent modes are calculated using the formula from [16, 17]

$$T_m = \alpha_m T_0 / (2m+1), \quad (14)$$

where T_0 is the Helmholtz mode period; α_m is parameter characterizing the basin form; m is mode number.

In [18], Table 2.1 with the periods of longitudinal modes of free oscillations in the basins of the simplest form is given. According to this table, in a semicircular-shaped basin with a depth profile specified by the dependence $h(x) = h_1(1\text{-}x^2/L^2)$, the ratio $\alpha_m/(2m+1)$ is equal to ~ 0.7, ~ 0.6 and ~ 0.5 for m equal to 1, 2 and 3, respectively. The Helmholtz mode period for such a basin is calculated by the formula

$$T_0 = 2{,}2 \cdot 2L \big/ \sqrt{gh_1}\ , \qquad (15)$$

where h_1 is depth at the basin inlet; L is its length equal to the basin water area radius.

We assume that the maximum in the spectrum of level fluctuations belongs to the Helmholtz mode, in this case T_0 = 47 min. Then the periods of the first, second and subsequent modes are 33, 28 and 24 min. Having determined the period of the most intense fluctuations (T_0) of free surface and knowing the basin depth at the inlet (h_1), it is easy to determine its length. Using relation (15), we obtain the expression $L = (gh_1)^{1/2}\,(T_0/4.44)$. Hence, the length of the basin L with a depth at its inlet $h_1 \sim 45$ m and the Helmholtz mode period T_0 = 47 min will be ~ 13.5 km.

The map-diagram of Posyet Bay (Fig. 1) demonstrates a semicircular water area with a diameter and depth at the inlet of ~ 28 km and ~ 45 m, respectively. According to Fig. 1, geometric dimensions of the model basin, as well as its shape and bottom profile, are in satisfactory agreement with the dimensions and shape of Posyet Bay in the first approximation.

In shallow bays and harbors, along with longitudinal fluctuations, there are also transverse seiche vibrations [19]. In what follows, we will need the periods of the first and subsequent modes of this type of oscillations. For the basin under consideration, the first mode period is calculated using the formula $\tau_1 = \tau_{max}/\sqrt{2}$. In this expression, $\tau_{max} = 8.88L/\sqrt{gh_1}$. Consequently, for the specified parameters of the basin, the period of the first transverse seiche τ_1 will be 70 min.

Thus, in the model of a semicircular sea basin with a quadratic bottom profile, the Helmholtz mode, the first and subsequent modes have periods of 47, 34, 29 and 24 min. In the same basin, the first and subsequent modes of transverse seiches have periods close to τ_1 = 70 min, τ_2 = 44 min, τ_3 = 31 min, τ_4 = 24 min.

We turn now to the analysis of the IWs frequency composition in area under study. Fig. 5 represents the spectrum of temperature pulsations caused by these waves. The spectrum is calculated within the 10–128 min range of periods, common with that of seiche vibrations. The numbers in the spectrum highlight its maxima, the periods of which are close to the ones of the maxima in the spectrum of free surface fluctuations of the bay, i.e. its seiche oscillations.

The calculations performed using the buoyancy frequency profile (Fig. 3) revealed that the phase velocity of the first-mode IW ranges within 0.15–0.3 $m \cdot s^{-1}$, and the wavelength λ_{in} with a period T_{in} ~15 min is ~ 300 m. Therefore, the bay is a shallow sea basin for IWs with the periods exceeding 15 min.

Next, we show that a sea basin with a depth of 45 m at the inlet is shallow for a surface wave with T_{sr} ~ 15 min period. The length of surface waves λ_{sr} (equal to $(gh_1)^{1/2}\ T_{sr}$) with this period is ~ 19 km, which significantly exceeds λ_{in}. Therefore, the bay is a sea basin in which the inequality $\lambda_{sr} \gg \lambda_{in} \gg H$ is satisfied, i.e. it is a shallow basin for both surface and internal waves with frequencies from the frequency range of seiche oscillations.

A sufficient condition for the "swinging" of the IW amplitudes with T_{int} period by seiche oscillations with T_{sr} period, taking into account (12), will take the following form:

$$\left|1-2T_{sur}/T_{int}\right| \le \mu / 2, \tag{16}$$

where $\mu = (u_0/c_{ph})$.

We are to show that IW amplitude with a phase velocity c_{ph} ~0.2 m·s^{-1} and a period T_{int} ~93 min is parametrically "swinging" by the Helmholtz mode with an amplitude ζ_0 ~ 0.1 m and a period T_{sur} ~ 47 min in the main resonance zone. For this purpose, we will check the sufficient condition for the implementation of this process. Condition (12) will be represented in the following form

$$\delta T/T_{int} \le \left(u_0/c_{ph}\right)\left(T_{int}/T_{sur}\right)/2, \tag{17}$$

where δT = (T_{int} − $2T_{sur}$) is a period mismatch; $u_0 = \zeta_0\sqrt{g/H}$ is the maximum velocity of barotropic current induced by the Helmholtz mode. Using the given values, we obtain $\delta T/T_{int}$ ~ 10^{-2}, u_0/c_{ph} ~ $2.5\cdot10^{-1}$. Thus, the right-hand side of relation (17) will be ~ 0.2, which is an order of magnitude greater than the left-hand side value of this relation. Consequently, the sufficient condition for the exponential growth of the wave amplitude with 93 min period and a phase velocity of ~ 0.2 m · s^{-1} is satisfied.

Now we check the sufficient condition under which the IW excitation with the frequencies of seiche oscillations of the bay is possible, i.e. parametric excitation of waves in the first zone of parametric resonance. We represent this condition in accordance with (13) in the following form

$$\delta T \le \left(u_0/c_{ph}\right)^2 T_{int}/2.$$

According to the works [16, 17], $u_0 = \eta_0\sqrt{g/H}$, then u_0 ~ 0.047 m·s^{-1}. Considering that c_{ph} ~ 0.25 m·s^{-1}, we obtain $\left(u_0/c_{ph}\right)^2$ ~ 0.035. Hence, the detuning of internal wave period with T_{int} = 47 min should not exceed 0.5 min.

It is obvious that verification of the sufficient condition (13) using the data of a natural experiment is a very complex task in terms of method. The relative stability of the excitation frequency of an internal wave with a period of 47 min, corresponding to the ratio $\delta T/T_{int}$, is ~ 1%, which is unlikely in marine conditions for the excitation interval, the upper limit of which is ~ 8 h.

At the same time, the excitation of internal waves in the first zone of parametric resonance is possible within the framework of the following scheme. Note that the periods of the most significant spectral maxima T_1, T_2, T_3 and T_4 are 17, 25, 29 and 47 min, respectively. The same periods correspond to 16, 26, 30 and 44 min of the semidiurnal tidal harmonics with a period of 12.4 h, which are close to the previous periods.

In [18], it was found that in Posyet Bay the tidal IW with a semidiurnal period changes its shape during propagation, i.e. the velocity of the liquid particles at the top exceeds the velocity of the particles at the bottom. In the spectral description of the wave motion this means that the maxima in the spectrum occur at $T_n = 12{,}4/n$ (h) periods, where $n = 1, 2, 3, \ldots$, is the harmonic number. Consequently, when standing surface waves with seiche oscillation frequencies propagate in a wave field, parametric resonance is possible between this field and the corresponding harmonics of the internal tidal wave with a frequency of 1/12.4 h^{-1}.

In other words, the semidiurnal tidal IW, propagating in the shallow water zone of the bay covered by seiche oscillations, transforms under the effect of quadratic nonlinearity from a harmonic wave with frequency $\nu = 1/12.4$ h^{-1} to a polyharmonic wave with harmonic frequencies $\nu_n = n\nu$. At close values ? between the frequencies of the seiche oscillations and those of the tidal IW harmonics, a parametric resonance occurs, i.e. an exponential increase in the amplitudes of the corresponding harmonics of the tidal IW.

Thus, in the presence of sufficiently intense seiche oscillations of the level and a weakly nonlinear IW with a frequency of $\nu = 1/12.4$ h^{-1}, a sufficient condition for the parametric generation of IW with the seiche oscillation frequencies in the first zone of parametric resonance is realized in the bay.

Conclusion

This paper examines the results of field studies of standing surface and free internal waves in Posyet Bay in the frequency range 1/16–1/256 min^{-1}. Using Fourier analysis, we identified the frequencies at which the most significant maxima in the spectra of both surface and internal waves are located in the specified frequency range. We have shown the proximity of a number of frequencies at which these maxima are located in the spectra of the specified wave processes.

Using a model basin approximating Posyet Bay, the period estimates of the Helmholtz mode and subsequent ones in such a basin were obtained. By analyzing the spectrum of the free surface fluctuations of the bay, we found that its maxima fall on the above periods, which are those of the level or seiche free fluctuations in the bay. Thus, the necessary conditions for the IW parametric instability caused by seiche level oscillations in the bay are formed in the autumn period.

Within the framework of the parametric resonance theory, it was found that under the influence of the barotropic current caused by seiche oscillations, the modulation of horizontal component of the IW orbital motion velocity takes place. With the corresponding ratio $u_0/c_{ph} \ll 1$, a sufficient condition for the parametric excitation of IWs in the zero zone of parametric resonance in the bay is realized.

Within the same theory, it has been shown that in the bays and coves of marginal seas, internal waves can be excited in the first zone of parametric resonance with frequencies of IW harmonics of $\nu_{td} = 1/12.4\ \text{h}^{-1}$ frequency. The condition for this resonance is that the frequencies of the Helmholtz mode and subsequent basin modes are close to those of the internal tide harmonics.

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