

## Resonance Generation of Short Internal Waves by the Barotropic Seiches in an Ice-Covered Shallow Lake

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*Purpose.* The observation measurements testify the fact that heat and mass transfer processes in the shallow ice-covered lakes are not limited to the molecular diffusion only. In particular, the effective thermal diffusivity exceeds the molecular one by up to a few orders of magnitude. Now it is widely accepted that the transfer processes, in spite of their low intensity, are controlled by intermittent turbulence. At the same time, its nature and generation mechanism are still studied insufficiently. The paper represents one of such mechanisms associated with resonance generation of short internal waves by the barotropic seiches.

*Methods and Results.* The temperature measurements in a shallow lake in winter were used as an experimental base. Having been analyzed, the temperature profiles' dynamics observed during a few weeks after freezing revealed the anomalous values of thermal diffusivity. At that the temperature pulsations' spectra clearly demonstrate the peak close to the main mode of barotropic seiches. Counter-phase oscillations at the different depths and pronounced heterogeneity of the amplitudes of temperature pulsations over depth indicate presence of internal waves. Based on these data, the mechanism of energy transfer from the barotropic seiches to the internal waves similar to the "tidal conversion" (the latter governs resonance generation of internal tides in the ocean), is proposed. The expressions for heat flux, energy dissipation rate and effective thermal diffusivity are derived.

*Conclusions.* Internal waves can play an essential role in the processes of interior mixing and heat transfer in the ice-covered lakes. Though direct wind-induced turbulence production is inhibited, baric perturbations in the atmosphere can give rise to barotropic seiches, which play the role of an intermediate energy reservoir and can generate short resonant internal waves resulted from interaction with the undulate lake floor. The internal wave field parameters strongly depend on the barotropic seiche amplitudes, buoyancy frequency and the bottom topography features.

**Keywords:** seiche conversion, ice-covered lake, temperature vertical profiles, internal waves, barotropic seiches, energy dissipation rate.

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### Introduction

During the winter, heat and mass transfer processes in shallow freezing lakes are rather slow and low energetic. At the same time, the nature of these processes, being very diverse and complicated [1], still remains an underdeveloped domain of



limnology. One challenging example is the dynamics of temperature vertical profiles during a few weeks after ice-on [2]. The values of effective thermal diffusivity, derived from profiles dynamics, usually exceed the molecular one several-fold [3] or even by a few orders of magnitude [2]. These estimates indicate directly that, while the process is not restricted to pure molecular transfer, it is not truly turbulent either. The nature of this additional transfer mechanism is the main topic of this paper. We concentrated on the study of waves of different nature, bearing in mind their possible impact on heat transfer via cascade processes and fine-scale mixing. Our study is based on experimental measurements carried out in a small shallow Lake Vendyurskoe (south of Karelia) during the ice-covered period.

Water movements in ice-covered freshwater lakes arise as a result of a number of factors, the main ones being river runoff, heat exchange with bottom sediments, air pressure disturbances over the lakes, and solar radiation penetrating under ice [1]. In this work, we discuss the experimental data obtained on the lake with insignificant river run-off in the period before the beginning of spring under-ice convection. Thus, heat exchange with bottom sediments and baric disturbances above the lake can be considered as the main acting factors.

As a result of heat exchange with bottom sediments, weak bottom currents directed from shallow water to deep-water parts of the basins are formed, while compensatory currents directed to shallow water develop in the surface layers [1]. A model calculation of such currents for Lake Vendyurskoe is given in [4]. It was shown that the velocities of such currents reach a maximum (more than 1 mm/s) in a thin 20-cm bottom layer in early winter and decrease by an order of magnitude during the winter, as the heat flux from the bottom sediments decreases.

The most probable origins of seiches in lakes throughout the winter are atmospheric pressure drop and wind exposure [5-8]. A review of experimental observations of level fluctuations and oscillating currents in ice-covered lakes is given in [5-8], a numerical calculation of seiches in such lakes is presented in [6, 9]. Seiche level fluctuations are clearly evidenced by the annual occurrence of ice cracks in the same places on large lakes - presumably in the antinodes of seiche (see the review in [6]).

Experimental confirmation of the existence of ice vibrations (with an amplitude of ~ 1-2 mm) and seiche-like currents (up to 3 mm/s) in the ice-covered Lake Vendyurskoe is given in [5, 7, 8]. As shown in these papers, the period of ice and current oscillations is close to a theoretical estimate of the first seiche mode. A quarter-period phase shift between the oscillations of ice and currents is consistent with the theory. Intensification of ice and currents oscillations in this lake occurs against rapid air pressure drops and strong winds. Thus, numerous experimental data and numerical calculations indicate the presence of barotropic seiches in the ice-covered lakes.

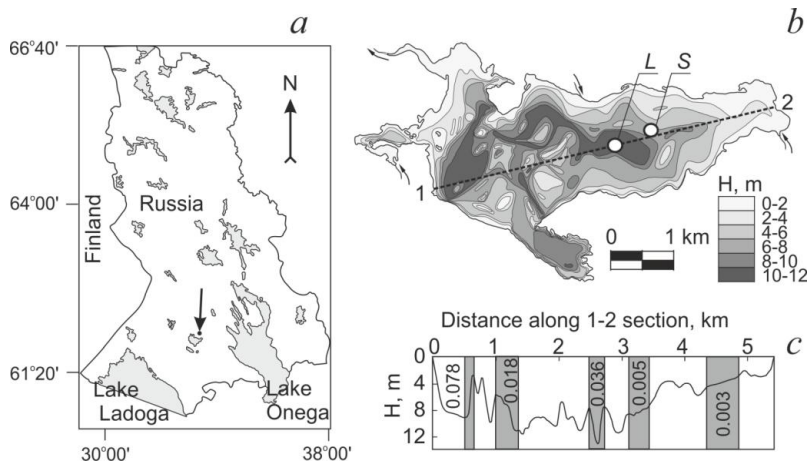
The first idea that fuelled this paper came from experimental data on current velocities: barotropic seiches are important hydrodynamic constituents in boreal lakes in winter, where they play the role of an energy pump. However, the velocities of the under-ice barotropic currents usually do not exceed a few mm/s [4, 7, 8], which is obviously not enough for such currents to be studied by methods commonly used in the theory of turbulence.

The next impulse for this study came from oceanological research on abyssal ocean mixing. A well-known solution of this problem is based on the ‘tidal conversion’ concept (e.g. [10–12]). It was suggested that the energy cascade is generated by long tidal waves interaction with undulating ocean floor, which leads to the generation of internal waves (IW). The interaction and nonlinear breakup of these waves govern the energy cascade to fine scales and finally to concomitant mixing and heat transfer enhancement.

We suppose that a similar mechanism works in ice-covered lakes, with barotropic seiches playing the role of tides. To the best of our knowledge, this ‘seiche conversion’ mechanism has never been discussed in the literature yet. Strong support for this hypothesis was obtained from the analysis of the high-precision temperature measurements carried out in a shallow ice-covered Lake Vendyurskoe, Karelia, Russia.

### Study site and measurements

Lake Vendyurskoe is a mesotrophic polymictic shallow lake of glacial origin (Fig. 1, *a*). The surface area of Lake Vendyurskoe is 10.4 km<sup>2</sup>, volume is 54.8 × 10<sup>6</sup> m<sup>3</sup>, maximal and mean depths are 13.4 and 5.3 m, respectively. The distribution of depths along the main axis of the lake is shown in Fig. 1, *b*, *c*. The ice season generally starts by late November-early December and ends by early to middle May; the maximal ice and snow cover thickness is 0.6–0.8 m [13].



**Fig. 1.** Location of the Lake Vendyurskoe at the map of Karelia (black dot on the arrowhead) – *a*; lake bathymetry and location of the measurement stations (white circles) – *b*; depths along the longitudinal section, bottom typical slopes are indicated (shaded fields) – *c*

Measurements of the water temperature were carried out in the ice season 2007–2008. Two thermistor chains (indicated in Fig. 1, *b* by letters *L* and *S*) were equipped with sensors *TR-1060 RBR Ltd.*, Canada (accuracy ± 0.002°C). The measurements were taken every minute. The sensors were spaced by 0.5–1.5 m within the water column. The depth at the locations of the chains *L* and *S* was 11.3 m and 7.1 m, respectively. We calculated the amplitudes of temperature oscillations  $\Delta T$  after subtracting the moving average (30 min) from the temperature series,

taking into account the 25–27 min period of the theoretical estimation of the first mode of the longitudinal barotropic seiche of Lake Vendyurskoe [14].

The effective thermal diffusivity  $K_T$ , averaged over a certain time interval  $t^*$ , can be estimated from the relationship between the thickness of the stratified layer and the temperature profile deformation time [2]:

$$K_T = (H - h_{mix})^2 / t^*. \quad (1)$$

Where  $H$  is the depth, m;  $h_{mix}$  is the mixed layer thickness, m;  $t^*$  is the duration of the period under consideration, s. The thickness of the mixed layer  $h_{mix}$  was estimated from the vertical temperature profiles in the central deep-water part of the lake ( $L$ -chain data) for the first three weeks of ice-period.

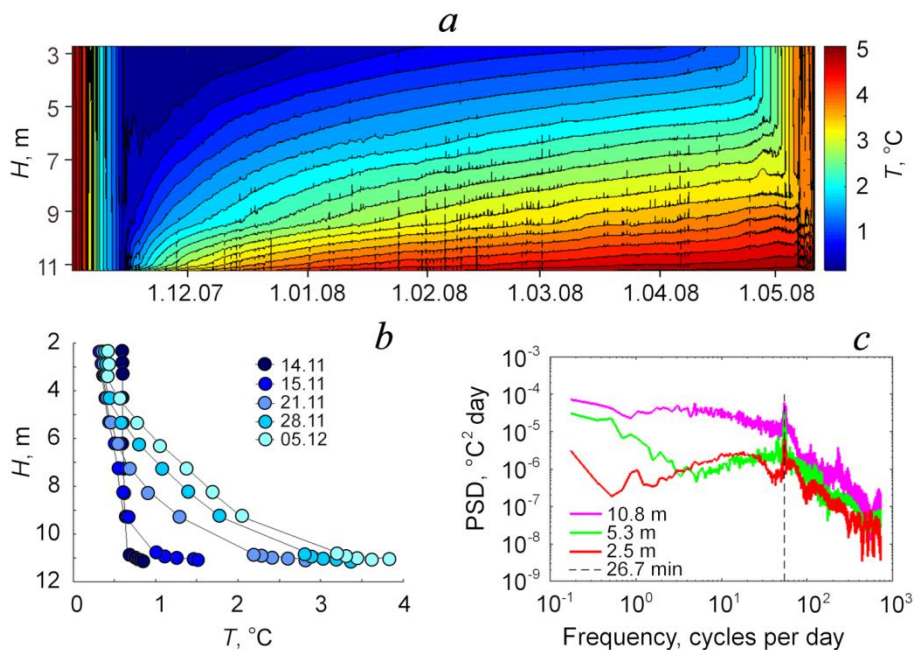
Spectra of temperature pulsations were obtained by the Welch method. Narrow band digital filters and moving averages were used.

### Results of measurements and preliminary analysis

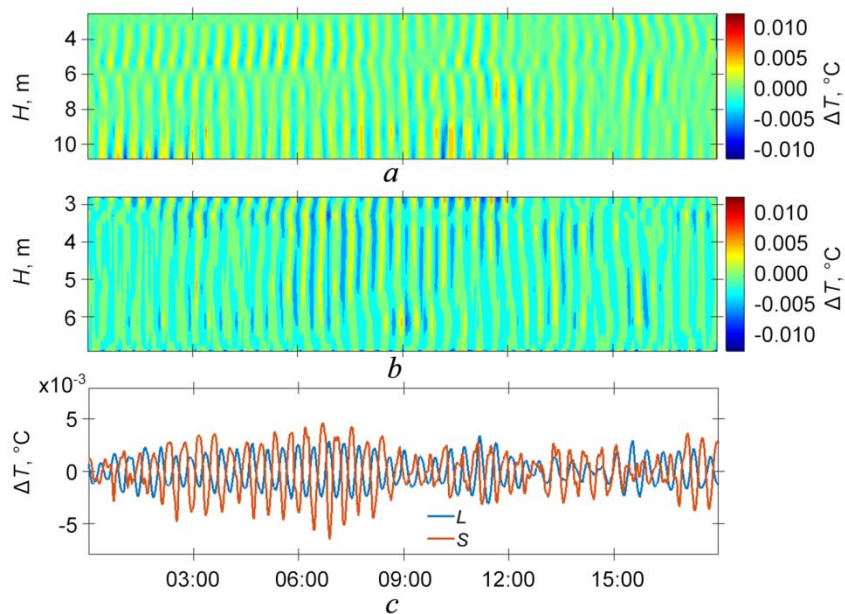
A continuous ice cover formed on the lake on November 14, 2007 at an average temperature of the water column of 0.55°C (Fig. 2, *a*). A few weeks after ice-on date, a homogeneous vertical temperature of the water column evolved fast towards a new, stably stratified, state (Fig. 2, *a*). A manifestation of the process is the formation of near-linear temperature profiles (Fig. 2, *b*) with a sharp jump in the bottom layer. For the deeper (5 and more meters) part of the water column, it usually takes only 3 weeks for such changes in the vertical profile to occur. The relatively fast transformation of the mean temperature profiles clearly indicates that the heat transfer is governed mainly by non-molecular mechanisms.

The values of  $K_T$ , estimated by (1), were  $1.4\text{--}3.6 \cdot 10^{-5}$  m<sup>2</sup>/s during first three weeks of 2007–2008 ice season (vs.  $1.3 \cdot 10^{-7}$  m<sup>2</sup>/s for molecular diffusivity of temperature). These estimates are in good agreement with the results obtained earlier for 1995–1997 and 1998–2000 early winter, when the values of  $K_T$  were  $\sim (2.3\text{--}8.8) \cdot 10^{-5}$  m<sup>2</sup>/s [2].

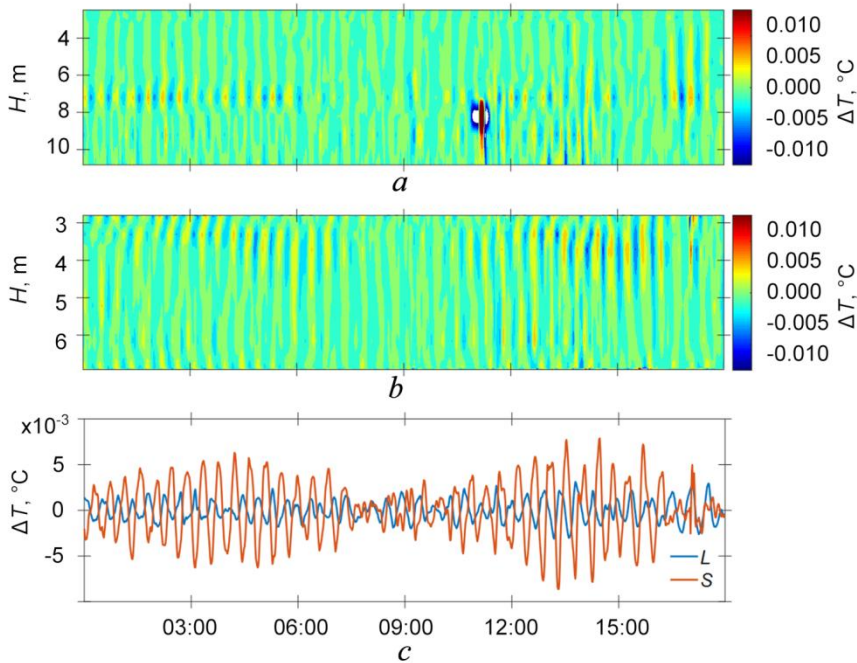
In the first weeks of ice-covered period, slope currents directed from shallow to the deep part of the lake can make a certain contribution to the enhancement of heat transfer. However, such currents quickly decay as heat flux decreases at the boundary of the water with bottom sediments [4], and cannot be the cause of a wide range of temperature pulsations, which were recorded throughout the water column at both measurement stations throughout the winter. The presence of alternative heat transfer mechanism is also manifest in the variability of the observed temperature pulsations. Pulsations spectra for some depths are presented by Fig. 2, *c*. Most pronounced peaks with period 26.7 min close to the main mode of barotropic seiche [14] were identified at both stations. The maximum at a certain (seiche) frequency at all depths (Fig. 2, *c*) indicates the presence of an energy source – a barotropic seiche. The heterogeneity of the amplitude of temperature pulsations at different depths indicates the presence of IW. This heterogeneity is further emphasized in Fig. 3 and 4. The variability of temperature pulsations at different depths (in particular, a noticeable increase at a depth of 10.8 m) is also due to (along with the fundamental reason – the presence of IW) the heterogeneity of the temperature gradient with maximum in the bottom layers of deep part of the lake (see Fig. 2, *b*).



**Fig. 2.** Water temperature in the central part of the Lake Vendyurskoe in winter, 2007–2008 (*L*-chain) – *a*; dynamics of vertical temperature profiles during the first three weeks of a freezing season – *b*; spectra of temperature pulsations for different depths during January 1–8, 2008 – *c*



**Fig. 3.** Water temperature dynamics (oscillations relative to the moving 30-minute mean) from 00 h 00 min to 17 h 59 min, January 10, 2008: *a* – at the *L*-chain, *b* – at the *S*-chain, *c* – at chains *L* and *S* on the depth 3.3 m



**Fig. 4.** The same as in Fig. 3; February 24, 2008

Most of the time series of water temperature at both chains and at different depths demonstrated periodicity, but the patterns were rather complicated, with strong temporal and spatial variability of oscillations (Fig. 3 and 4). This behaviour resembles the effects of interaction, instability, and breakup of IW with different frequencies and phases. The breakup of IW and their interactions seem to be a rather probable source of fine-scale pulsations.

In order to interpret and describe the complicated wave patterns, particular attention was given to the spatial (especially vertical) structure of temperature oscillations. The analysis revealed the presence of short IW [15]. This conclusion was triggered by the estimations of the amplitudes  $\Delta T$  of temperature oscillations (after subtracting the 30-min. moving average). These amplitudes, as data analysis proves, vary significantly with time and depth. For example, time intervals with  $\Delta T$  background values around 0.001 K alternate with active ones, where  $\Delta T$  is an order of magnitude higher or even more (Fig. 3, 4).

Within standard assumptions, one can regard temperature pulsations and their amplitudes  $\Delta T$  to be indicators of vertical displacements  $s = \Delta T / (dT/dz)$  of water layers, and here comes the first surprise. Taking into account the typical value 0.3 K/m for the mean temperature gradient  $dT/dz$ , a value at around 2–3 mm for  $s$  directly follows for background oscillations. This value is quite comparable with typical displacements of water and ice by barotropic seiches [7]. On the other hand, the above-mentioned active time intervals correspond to water displacements up to 10 cm and more. Such high values are practically ruled out within the barotropic oscillation scenario, so one can suppose that IW are in charge.

To test this hypothesis, we studied the dependence of temperature pulsation amplitudes on the depth  $z$  at both chains and in different periods of the ice season.

The results varied among dates and locations, but in most records, a close dependence of  $\Delta T$  on  $z$  was observed. In some cases, this vertical inhomogeneity was particularly strong, manifest in a vertical splitting of the water column into several layers with increased and suppressed levels of oscillations. At that the oscillations in the adjacent active levels were counter-phased. Fig. 3, *a, b* and 4, *a, b* demonstrate the above-described high-mode vertical structure of the oscillations. In fact, high vertical modes are typical only for IW, so the revealing of such modes clearly proves the baroclinic nature of oscillations.

The detection of IW was just a recovery of the missing link in establishing the seiche conversion mechanism of energy transfer, similar to the oceanic tidal conversion concept mentioned above.

### **Seiche/tidal conversion. Assumptions and some applications**

To identify the mechanisms of mixing and enhancement of heat transfer in an ice-covered lake, it seems reasonable to emphasize the main physical features, namely the stable water stratification and the presence of an oscillating background flow (barotropic seiche with a basic frequency  $\omega_0$ ).

The same features are typical for abyssal ocean, where barotropic tides play the role of a background flow. A clear physical analogy is readily observed. Moreover, the difference between the typical values of the key parameters in the two cases is not crucial. The amplitude  $U_0$  of the flow velocity and the buoyancy frequency  $N$  in the ocean are usually estimated at around 0.1 m/s and  $10^{-3} \text{ s}^{-1}$ , whereas the values of these parameters in lakes lie within (0.001–0.01) m/s and  $(10^{-3}–10^{-2}) \text{ s}^{-1}$ .

On the other hand, the problem of abyssal ocean mixing is well-established and a solution that has been widely accepted is to address the interaction of oscillating flows with bottom topography. This mechanism is known as tidal conversion and explains the transfer of the basic flow energy to IW, which eventually feed the fine-scale motions and mixing.

Considering the above analogy, it is reasonable to apply the results of the tidal conversion approach to the case of ice-covered lakes. The energy flux is the issue of primary interest and importance here because its value determines the capability of IW to enhance the heat transfer.

In the oceanographic literature, this issue is discussed in a large number of papers covering a wide range of cases corresponding to different topographies. In general, the calculations are rather complicated and even sophisticated; suffice it to say that the problem includes at least six parameters [16], and usually some special assumptions are necessary. Thus, the application of the results to the case of lakes is not straightforward and requires a critical and selective overview and rearrangement.

To this end, it is reasonable to start with the analysis of the simplest results, which were obtained within the tidal conversion theory under the most refined problem statement [11, 12]. This statement usually includes the following assumptions:

I) the tidal excursion distance  $U_0/\omega_0$  is much smaller than the horizontal scale of the topography  $l$ ;

II) weak-topography approximation: topographic slopes are much smaller than the slope  $\mu$  of a tidal beam

$$\mu = \frac{k}{m} = \sqrt{\frac{\omega_0^2 - f^2}{N^2 - \omega_0^2}}.$$

(here,  $k$  and  $m$  are the horizontal and vertical wave numbers, respectively;  $f$  – Coriolis parameter);

III) the reflection from the upper surface is negligible.

In most papers, it is also assumed that the buoyancy frequency  $N$  does not depend on the vertical coordinate  $z$ , and the bottom topography varies only in the direction  $x$ , so all physical fields are planar. These two last assumptions are mostly technical, introduced to simplify the calculations. The approaches to overcome these restrictions were discussed in some recent papers (e.g. [12]). As for the frequency  $f$ , in the lake case it is negligible as compared with both the fundamental frequencies  $N$  and  $\omega_0$ . The estimation  $R \sim HN/f$  of the Rossby radius  $R$  for Lake Vendyurskoe gives the value  $\sim 500$  m, which is far above the typical IW wavelengths [15]. So, Coriolis effects are hereafter neglected.

Given all these assumptions, direct analytical calculations of the tidal conversion from the isolated smooth hill  $h(x)$  become possible. With respect to the assumption I, one can neglect Doppler-shifting effects and regard the IW spectra as concentrated on the basic frequency  $\omega_0$  [17]. In this case, the wave amplitude is sensitive only to the slope of the obstacle [11] and the expression of the vertical energy flux or conversion rate  $W$  ( $W$ ) takes the form

$$W = \rho_0 U_0^2 L \sqrt{N^2 - \omega_0^2} \int_0^\infty kh(k)h^*(k) \frac{dk}{2\pi}. \quad (2)$$

Here,  $\rho_0$  is the reference water density,  $h(k)$  is the Fourier transform of  $h(x)$ ,  $*$  means the complex conjugate,  $L$  is the length of the hill in the direction perpendicular to the main flow velocity. For simple sinusoidal topography  $h(x) = h_0 \cos(k_0 x)$ , calculations are straightforward and the following expression for the density of the energy flux  $J$  ( $W/m^2$ ) is derived from (2) [18]:

$$J = Y(\varepsilon^*) \rho_0 U_0^2 h_0^2 k_0 \sqrt{N^2 - \omega_0^2} / 4. \quad (3)$$

Here,  $Y(\varepsilon^*)$  is a non-dimensional function of the parameter  $\varepsilon^* = k_0 h_0 / \mu$ . The weak-topography approximation (see assumption II) implies the strong inequality  $\varepsilon^* \ll 1$ ; in this case,  $Y$  is close to unity.

A similar result is valid in a more general case, where the bottom topography is modelled as a population of hills with height  $h$  and length  $l$ :

$$J = C \rho_0 U_0^2 \sqrt{N^2 - \omega_0^2} h^2 / l. \quad (4)$$

Here,  $C \sim 1$  is the numerical constant, which depends on the specific profile of the hill.

Although the assumptions listed above are rather rough and numerous, the estimates of energy flux density based on (3) and (4) match fairly well



the observed oceanographic data. In particular (in qualitative accordance with early Bell's predictions  $J \sim 10^{-3}$  [19]), the calculated values  $J \sim 10^{-2}$  W/m<sup>2</sup> for the Hawaiian Ridge [12] and  $3\text{--}5 \cdot 10^{-2}$  W/m<sup>2</sup> for the Mid-Atlantic Ridge in the South Atlantic [20] seem to be quite consistent with the available experimental estimates.

Most authors also stress the high horizontal inhomogeneity of the energy flux, so its average value across the ocean is reduced by at least two orders ( $\sim 10^{-5}$  W/m<sup>2</sup>) [10]. Recent experimental estimates produce the value  $J \sim 10^{-5}$  W/m<sup>2</sup>, which is now regarded as the general (averaged) background level of mixing for the ocean interior [21].

### IW energy flux and dissipation rate $\varepsilon$ for Lake Vendyurskoe.

Before transferring the estimates (2) – (4) to the case of lakes, it is necessary to check the applicability of the assumptions. The typical values of the significant parameters are presented in Table.

**Typical values of the oscillating currents' parameters in lakes and oceans**

Basin	$U_0$ (m/s)	$\omega_0$ (1/s)	$N$ (1/s)	$U_0/\omega_0$ (m)	Bottom slope	$\mu$	$\varepsilon^*$
Ocean	0.01–0.1	$10^{-4}$	$10^{-3}$	$\sim 100$	$\sim 10^{-3}$	$\sim 0,1$ (for latitude 30°N)	$\sim 0.01$
Lake	$10^{-3}\text{--}10^{-2}$	$10^{-2}$	$10^{-2}$	$\sim 1$	$\sim 10^{-2}$	$\sim 0.3$	$\sim 0.03$

As for Lake Vendyurskoe, the period of the first-mode barotropic seiche is approximately 27 min [14]; see also Fig. 2, *c*. So with  $\omega_0 \sim 4 \cdot 10^{-3}$  s<sup>-1</sup> and  $U_0 \sim (1\text{--}10)$  mm/s [9] the seiche excursion distance is only a few meters long. On the other hand, the estimates of  $l$  from the lake bottom topography map (Fig. 1, *c*) yield values at around a few hundreds of meters. Thus, assumption I is applicable for the lake. Assumption II is also valid after taking into account  $N \sim 10^{-2}$  s<sup>-1</sup> and  $\mu \sim 0.3$  (see Table). To check assumption III, IW wavelengths estimations are necessary. This issue is discussed below.

Now we can propose a very rough estimation of the energy flux in lakes. We can model the bottom topography as a population of hills with a typical height  $h \sim (2\text{--}3)$  m (Fig. 1, *c*). As for the average slope, the following general estimation was used [22],

$$\frac{h}{l} \approx \frac{D}{0,165\sqrt{A}}. \quad (5)$$

where  $D$  is the median depth and  $A$  is the lake area. After setting the median depth equal to the mean depth  $H$  (5.3 m), the simple calculations by (5) lead to the value  $h/l \sim 0.01$ . The direct estimations carried out with experimental bathymetry profiles for Lake Vendyurskoe (see Fig. 1, *c*) give a close value of 0.016.

Finally, taking (5) into account and substituting the typical values  $10^{-3} - 10^{-2}$  m/s,  $10^{-2}$  s $^{-1}$  and 2 m for  $U_0$ ,  $N$  and  $h$ , respectively, in (4), the following values of the upward energy flux are readily obtained (W/m $^2$ ):  $J \sim 5 (10^{-7} - 10^{-5})$ .

These values are of orders smaller than the above mentioned values for the ocean; but such a result was expected, considering the difference for the typical values of  $U$  and  $h_0$  in the ocean and lakes.

The estimation  $J$  of the energy flux is the basis for mixing efficiency and eddy diffusivity evaluation. To begin with, it seems reasonable to analyze this estimation in a broader context.

Namely, the flux value  $J$  makes it possible to suggest a direct estimation of the decay time  $\tau$  of the background flow (seiches). This flow plays the role of an energy reservoir for IW, but the latter, in turn, reduce the density  $K = \rho_0 U_0^2 / 2$  of the kinetic energy of the main flow with time. Then, following the elementary energy balance  $J\tau \sim KH$  for the water column, substituting the typical values of  $U_0$  and  $H$ , the estimate for the seiche decay time is easily obtained:  $\tau \sim 10^5$  s. This value (a few days) is consistent with previous estimations for closed basins [23].

Taking (4) into account, a more general result can be easily derived from the energy balance:

$$\tau \sim \frac{Hl}{h^2 \sqrt{N^2 - \omega_0^2}}. \quad (6)$$

According to (6), the seiche decay time depends on  $H$ ,  $N$ , and on the parameters  $h$  and  $l$  of the bottom topography.

As for dissipation rate  $\varepsilon$  estimations, the task of deriving the value of  $\varepsilon$  from estimation for  $J$  is not straightforward. Even with high values of energy conversion rates, difficult questions arise concerning the mechanisms of IW steepening and breaking and final energy transfer to small scales [24–26]. Usually, the Richardson number of IW is large, but the effect of beam collimations corresponding to the bottom areas with the maximum slopes may be in charge of wave breaking [18, 25]. With this scenario, one can expect a sporadic and intermittent character of turbulence, with high sensitivity to the details of bottom microstructure. Besides, the reflection of waves from the lower ice boundary must be taken into account.

In this paper, we avoid discussing these problems in detail, but assume that for a stationary regime the radiated energy is eventually transferred to fine scales. Then, as a roughest approximation for  $\varepsilon$ , we can explore the expression for energy production  $J/(\rho_0 H)$  per unit mass. In this case, the estimation

$$\varepsilon \sim \frac{U_0^2 N h_0^2}{lH} \sim (10^{-10} - 10^{-8}). \quad (7)$$

is straightforward ( $[\varepsilon] = \text{m}^2/\text{s}^3$  or W/kg).

This estimate is now not so striking: despite the substantial difference of  $J$  between lakes and the ocean, the values of  $\varepsilon$  in both cases are quite similar (for the abyssal ocean  $\varepsilon \sim 10^{-10} - 10^{-9}$  [27]). The estimate (7) also conforms to the lower boundary of the values obtained for lakes in other papers [28–30].

To achieve the preliminary qualitative characteristics of the heat transfer enhancing mechanism, it is reasonable to compare the obtained values of  $\varepsilon$  with the well-known ‘active’ turbulence criterion  $\varepsilon > 20\nu N^2$ , or  $Re_b > 20$  [30, 31] (here,  $\nu$  stands for molecular viscosity,  $Re_b = \varepsilon/(\nu N^2)$  is the so-called buoyancy Reynolds number). Due to this criterion, for  $N \sim 10^2$  the threshold value  $\varepsilon_{cr}$  is limited by  $\varepsilon_{cr} > 2 \cdot 10^9$ . It means that the energy of the generated IW is insufficient for invoking fully developed turbulence in the entire bulk of the water column. This conclusion aligns with the common understanding of turbulent mixing in the interior of lakes and oceans, i.e., that the mixing occurs due to sporadic and localized shear instability events [30]. So, the mixing enhancing mechanism can be described as sporadic and intermittent turbulence.

### Typical time and spatial scales. Mixing and effective diffusivity

Moving on to quantitative description of the mixing mechanism, it is advisable to start with estimation of the basic time and space scales relevant to heat and mass transfer. The range (7) of  $\varepsilon$  values derived in the previous section makes these estimations straightforward. In particular, for Kolmogorov scales of length  $r_K = (\nu^3/\varepsilon)^{1/4}$ , velocity  $v_K = (\nu\varepsilon)^{1/4}$ , and time  $t_K$  we easily obtain:

$$r_K \sim (1-3)10^{-3}; v_K \sim (1-3)10^{-4}; t_K \sim 10.$$

Next, the Ozmidov scale  $L_O = (\varepsilon/N^3)^{1/2}$ , characterizing the size of energy containing eddies, is of interest. Direct calculations yield the values  $L_O = 3 \cdot 10^{-3} \div 3 \cdot 10^{-2}$ , which correspond to the lower limit of this scale for small, strongly stratified lakes [30]. As the  $L_O$  scale sufficiently (several-fold) exceeds the Kolmogorov scale, one can expect to have a range of scales in which a turbulent cascade might be present [32].

To obtain the estimate of the effective eddy diffusivity, we use the well-known Osborn relation [33]:

$$K_T \sim \Gamma \varepsilon / N^2. \quad (8)$$

Here, the so-called mixing coefficient  $\Gamma$  is expressed in terms of the mixing efficiency  $\eta$ :  $\Gamma = \eta/(1-\eta)$ . The parameter  $\eta$ , usually referred to as the flux Richardson number, is the ratio of the mixing-induced potential energy gain to the turbulence production [34, 35]. The value of  $\eta$  was derived from observational measurements for both oceanic [34, 36] and limnic [30] environments. As was stressed in most papers, the parameter  $\eta$  is not constant, depending in particular on the evolution of the turbulent event [34]. Besides, in lakes these events are intermittent in space and time, thus complicating the derivation of the average value of  $\eta$ . On the other hand, the approximate value  $\eta \sim 0.17$  [36], consistent with the main bulk of observational data, is usually accepted for most practical applications.

Substituting  $N \sim 10^2 \text{ s}^{-1}$  and the estimation (7) for  $\varepsilon$  in (8), we obtain the following result for the range of eddy diffusivity:  $K_T \sim (10^{-7} - 10^{-5}) \text{ m}^2/\text{s}$ . These estimates exceed the molecular value by up to two orders and qualitatively conform to the experimental data described in Section ‘Results of measurements and preliminary analysis’. It is also worthwhile to compare the derived values of  $K_T$  with

the lower limit  $10^{-6} \text{ m}^2/\text{s}$  of eddy diffusivity in lakes in the open water case [30]. Substituting  $N \sim 10^{-2} \text{ s}^{-1}$  and the estimation (7) for  $\varepsilon$  in (8), we obtain the following result for the range of eddy diffusivity:  $K_T \sim 10^{-7} - 10^{-5} \text{ m}^2/\text{s}$ . These estimates exceed the molecular value by up to two orders and qualitatively conform to the experimental data described in Section ‘Results of measurements and preliminary analysis’. It is also worthwhile to compare the derived values of  $K_T$  with the lower limit  $10^{-6} \text{ m}^2/\text{s}$  of eddy diffusivity in lakes in the open water case [30].

### Discussion

All the results above were derived with respect to assumptions I–III. As was demonstrated, the assumptions I and II are quite reasonable. Assumption III needs a special consideration, because for the case of a finite-depth body of water, the reflected waves must be taken into account. With the new (upper) boundary conditions, the calculations become more complicated, but the final result for the energy flux still preserves the basic features of the expression (2). The only, although very essential, difference concerns the IW wavenumbers. Namely, only the resonant set

$$k_j = \frac{j\pi}{H} \frac{\omega_0}{\sqrt{N^2 - \omega_0^2}}, \quad j = 1, 2, \dots$$

of wave numbers must be taken into account in (2) [12, 37]. It means that the conversion rate is very sensitive to topographic roughness with horizontal scales corresponding to the mentioned set. As stressed by Jayne et al. [21], the modelling of undulating ocean floor by smooth profiles (with the small contribution from harmonics  $\{k_j\}$  responsible for IW generation) underestimates the true conversion by almost an order of magnitude.

So, floor profiles with higher spatial resolutions are necessary for better prediction of the conversion rate for a finite-depth body of water. High resolution bathymetry data are hardly available, but one can gain information from some general results concerning the surface topography. The numerous studies of terrestrial and fluvial landscapes revealed their fractal nature over a wide range of scales from 0.1 m to  $10^5$  m [19, 38]. It means, in particular, that the coefficients  $h(k_j)$  in the Fourier decomposition of  $h(x)$  are scaled as  $h(k_j) \sim k_j^{-\beta}$  with increasing  $k$ . The value of the scaling factor  $\beta$  estimated from power-law topography spectra is close to  $2/3$ .

Taking into account the fractal nature of topography, we can adjust the estimation (4) to the case of finite depth. The topographic factors in (4) are represented by the product  $h^2/l$ , where  $h$  is the typical height of a hill with the horizontal scale  $l$ . The value of  $J$  was estimated for  $l \sim 100$  m, in accordance with available bathymetry data. On the other hand, the resonant horizontal wavelengths

$$\lambda_j = \frac{2H}{j} \frac{\sqrt{N^2 - \omega_0^2}}{\omega_0}$$

for Lake Vendyurskoe are defined by the sequence  $\sim (30, 15, \dots)$  m. Statistically reliable topographic data for such scales are not available, but indirect estimations are straightforward. Indeed, taking the landscape scaling into account, the following relation is readily obtained:

$$h^2(\lambda_j)/\lambda_j \sim (h^2/l)(\lambda_j/l)^{2\beta-1}.$$

This relationship means that the input to the energy flux from the first resonance mode is proportional to  $(\lambda_j/l)^{1/3}$ . This complementary factor is simply the coefficient reducing the value  $J$  of the energy flux estimate. For the main ( $j = 1$ ) IW mode the ratio  $\lambda_j/l$  is approximately 0.3, but due to the small power ( $2\beta - 1 = 1/3$ ) this reduction is not crucial, and the main conclusions remain valid.

Further bathymetry studies with higher resolutions are planned in order to obtain the micro roughness spectra and more precise estimations of the basic heat transfer parameters.

### Conclusions

The article discusses one of the mechanisms of heat transfer enhancement in ice-covered lakes. The essence of this mechanism, similar to the tidal conversion mechanism existing in oceans, can be represented by the following energy cascade chain: atmosphere baric perturbations – barotropic seiches – short IW – fine-scale dissipative motions. The analysis of observational data from Lake Vendyurskoe proves the presence of resonant IW. The energy flux induced by these waves strongly depends on barotropic seiches activity and bottom topography, varying in a wide range  $5 (10^{-7} - 10^{-5})$  W/m<sup>2</sup>. Taking into account the universal statistical properties of terrestrial landscapes, for future work it seems reasonable to study the general correlation between winter stratification dynamics and bottom morphometric characteristics for different lakes. The energy dissipation rates and the coefficient  $K_T$  of thermal turbulent diffusion are estimated. Although highly intermittent, the internal wave degradation and breaking events may lead to an overall increase of the  $K_T$  value by up to two orders of magnitude as compared with the molecular limit.

Verification of the estimates of the effective thermal diffusivity coefficient for the late winter period is supposed to be carried out on the basis of studying the correlations between the dynamics of ice growth and the balance of heat fluxes on its lower edge.

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