

## Basic Regularities of Vertical Turbulent Exchange in the Mixed and Stratified Layers of the Black Sea

A. S. Samodurov ✉, A. M. Chukharev, D. A. Kazakov

*Marine Hydrophysical Institute of RAS, Sevastopol, Russian Federation*

✉ [anatol\\_samodurov@mail.ru](mailto:anatol_samodurov@mail.ru)

**Purpose.** The purpose of the study is to assess the coefficient of vertical turbulent exchange for different layers of the Black Sea basin based on the experimental data on microstructure of the physical fields obtained for the period 2004–2019 in the Black Sea and using the semi-empirical models.

**Methods and Results.** For the upper mixed layer, the turbulent energy dissipation rate  $\epsilon$  and the exchange coefficient were calculated using the velocity fluctuation spectra based on the Kolmogorov hypotheses on the turbulence spectrum inertial range. In the stratified layers, the turbulence coefficient and the dissipation rate were experimentally determined both from the spectra of the velocity horizontal fluctuations' gradients and the vertical spectra of temperature fluctuations using the concept of the effective scale of turbulent patches. Depending on the features of the hydrological regime and the prevailing energy contributors to turbulence generation, five layers were identified and described (including their characteristic power dependences of the vertical turbulent diffusion coefficients  $K$  on the buoyancy frequency  $N$ ) using the 1.5D-model of vertical turbulent exchange for the basin under study. For the stratified layers, the 1.5D-model results were comparatively analyzed with those of the other semi-empirical and theoretical models describing the most probable hydrophysical processes in each specific layer; the relations for the vertical turbulent exchange coefficient were obtained depending on the buoyancy frequency.

**Conclusions.** Comparison of the experimental data collected under different hydrometeorological conditions with the simulations resulted from the known turbulence models for the sea upper layer showed that the best agreement between the simulation and measurement data was provided by a multiscale model taking into account three basic mechanisms of turbulence generation: current velocity shear, instability of wave motions, and wave breaking. The turbulent exchange coefficient dependencies on depth are conditioned by the effect of the turbulence dominant source at a given level. In the stratified layers, the exchange coefficient dependence on buoyancy frequency is determined by the hydrophysical processes in each layer; the relations obtained for individual layers indicate intensity of the contributions of vertical advection, internal wave breakings, turbulence diffusion and geothermal flux.

**Keywords:** Black Sea, energy dissipation, stratified layer, vertical turbulent exchange, buoyancy frequency, measuring complex, field measurements, turbulent exchange models, 1.5D-model

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### Introduction

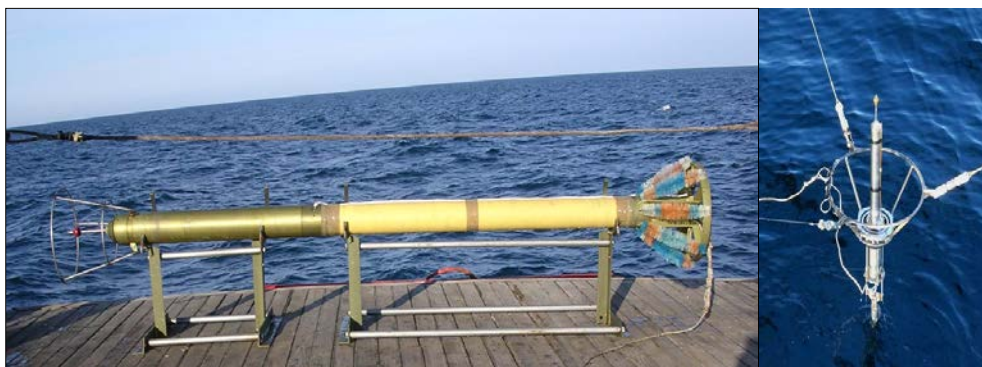
Over the past decades, the need for predictive assessments of the functioning of the oceans and seas ecosystems and the solution of a wide range of scientific and practical problems associated with this has increased. For this purpose, it is necessary to use the data on the nature and intensity of physical exchange

processes in the sea active layer to create adequate practical calculations models of the nutrients transfer, pollutants from submarine wastewater discharges and accidental emissions, to assess the deep-sea layers ventilation activity, etc. All of these processes depend in the most essential way on turbulent exchange, which, in its turn, is determined by the hydrological regime and can vary widely in time and space. Revealing the regularities that determine the distribution of vertical turbulent exchange coefficients is a necessary condition for objective assessments of the intensity of various substances vertical flows. These regularities can be further used to solve a wide range of oceanological problems.

The purpose of this work is to generalize the long-term results of experimental and theoretical studies of turbulent regime in various layers of the Black Sea in order to give an integral picture of turbulent exchange intensity dependence on environmental conditions and physical processes affecting the vertical mixing.

### Experimental data

The collection of experimental data was carried out using a specialized measuring complex “Sigma-1”, developed at Marine Hydrophysical Institute and designed for studying the microstructure of hydrophysical fields [1]. The complex allows registering a wide range of physical characteristics: current velocity fluctuations, temperature and electrical conductivity (including their fluctuation values), pressure. The appearance of the complex various modifications is demonstrated in Fig. 1. The studies in the upper stratified layers were carried out during expeditionary work at R/V in 2004–2019. When performing the probing, the instrument freely moves downward at a constant velocity of approximately 0.75 m/s. The nature of movement is recorded by the built-in accelerometers system, which makes it possible to estimate the natural oscillations of the carrier. Such control is necessary to ultimately achieve “pure” data on the velocity fluctuations in the medium. In this case, the carrier's own movements influence on the measurement results is eliminated by appropriate data processing.



**Fig. 1.** Measuring facilities "Sigma-1": on the left – for studying the hydrophysical fields' microstructure (probing version); on the right – for studying turbulent processes in the near-surface layer (positional version)

When studying the turbulent regime in the homogeneous upper layer, the positional version of “Sigma-1” is installed on a stationary oceanographic

platform using a specially designed system<sup>1</sup>. In this case, measurements of turbulent fluctuations are supplemented with information on the mean current, surface waves and meteorological data.

As an objective assessment of the turbulence intensity, as a rule, we used the turbulent energy dissipation rate values, which were determined differently for the positional and probing measurement options. In the first case, the rate of turbulent energy  $\varepsilon$  dissipation was calculated by the method proposed in [2] and then described in [3]. At the same time, signal distortions caused by suspension device swell and vibrations do not significantly affect the result. The method is based on Kolmogorov's hypothesis, according to which the velocity fluctuations spectral density  $E$  can be represented as

$$E(k) = \varepsilon^{1/4} \nu^{5/4} F(\lambda),$$

where  $k$  is a wave number;  $\nu$  is a kinematic viscosity;  $F(\lambda)$  is a universal function (model spectrum  $\lambda = k / (\varepsilon^{1/4} \nu^{-3/4})$ ) is a dimensionless wave number.

Energy dissipation rate was determined by combining the velocity fluctuations experimental spectrum with the model one, which was the Nasmith spectrum [4, 5]. Our data comparison, namely, the calculated values of the dissipation rate when normalizing to the energy flux from the atmosphere to the waves, with the results of *in situ* measurements carried out using the WAVES research programs [6] on Lake Ontario and SWADE [7] in the North Atlantic, where the characteristics of turbulence near the air – water surface were studied, showed their good agreement.

In the second case, the dissipation rate was determined using the technique described in [8] with regard to *MST (Microstructure-Turbulence) Profiler* instrument equipped with *PNS 93* gauge for measuring the velocity shifts.

The main difference of “Sigma-1” complex from the specified instrument is that the device directly measures the velocity fluctuations, separates the horizontal component and calculates the smoothed values of the speed fluctuation gradient  $\Delta u / \Delta z$  in a certain layer, according to which the turbulent energy dissipation rate is determined. In this case, vertical fluctuations are not quite informative due to the high vertical velocity of the instrument; therefore, a vertical gradient of horizontal fluctuations is used. For this purpose we first remove outliers and noise emissions due to natural oscillations of the instrument, and then use a band-pass filter to restrict the measured inertia frequency fluctuations by the subband of the turbulence spectrum from the low-frequency side ( $k_l$ ) and the Kolmogorov wavenumber

$$k_c = (1 / 2\pi)(\varepsilon / \nu^3)^{1/4}$$

at the upper boundary where  $\nu$  is a kinematic viscosity.

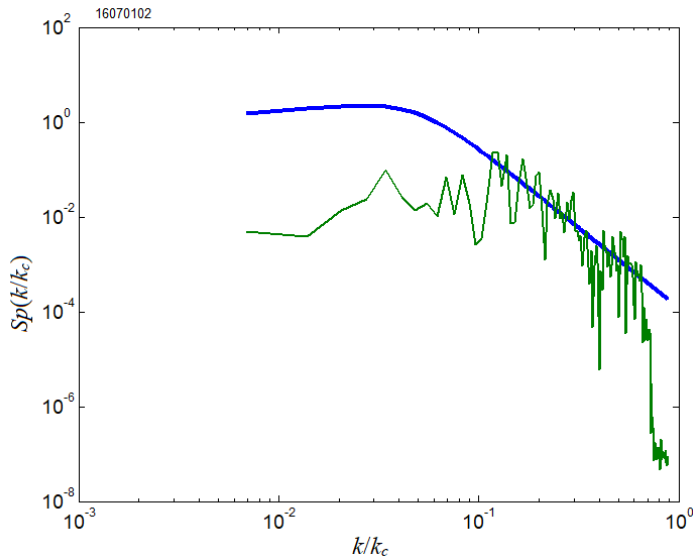
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<sup>1</sup> Barabash, V.A., Samodurov, A.S., Chukharev, A.M., 2015. [*Measuring System for the Study of Small-Scale Turbulence in the Near-Surface Layer of the Sea*]. Patent RF, no. 2014151917/93.

The dissipation rate is calculated by the iterative method: firstly, the boundary wave numbers  $k_l$  and  $k_{max}$  are determined, then the spectrum of  $du/dz$  value is calculated by the Welch method (P. D. Welch). In this case, the records are divided into overlapping segments, which are multiplied by the Hann time window, then we perform the Fourier transform, followed by averaging the spectral functions over all segments. The dispersion  $\left[ \overline{(du/dz)^2} \right]$  is determined by integrating the spectrum values in the selected range of wavenumbers. The dissipation rate of turbulent energy is determined by the relation

$$\varepsilon = \frac{15}{2} \nu \left[ \overline{(du/dz)^2} \right].$$

This  $\varepsilon$  value is used to calculate the Kolmogorov wave number. If the stop conditions are not met, the cycle is repeated starting from the spectrum calculation. The stopping criteria are a small variation of  $k_c$  value (less than a step in  $\Delta k$  spectrum) and an excess of  $k_c$  over  $k_{max}$ . An example of the calculated dimensionless spectrum in 72–75 m depth range is represented in Fig. 2. For comparison, the same figure demonstrates the Nasmyth Universal Spectrum [4].



**Fig. 2.** Comparison of the Nasmyth Universal Spectrum (blue line) and the experimental spectrum of the horizontal fluctuations' gradients (green line) at the 72–75 m depth. The fluctuations were preliminary smoothed by the median filter and subjected to a band-pass filtering. The dissipation rate is  $5.3 \cdot 10^{-8} \text{ m}^2 \cdot \text{s}^{-3}$

In order to estimate the coefficients of vertical turbulent diffusion and turbulent energy dissipation rate, a method based on the energy analysis of the evolution of turbulent (mixed) patches was also used. The effective scale of turbulent patches was estimated from measurements of temperature fluctuations and the vertical spectra calculated from them. The method is convenient when the density gradient is mainly determined by the temperature gradient contribution.

In [9, 10], it was developed a method for calculating the dissipation rate  $\varepsilon$  dependence on the buoyancy frequency  $N$  using the effective scale of turbulent patches  $L$  and the dependence of vertical turbulent diffusion coefficient  $K$  on  $N$ :

$$\varepsilon \cong 0.1L^2N^3, \quad K \cong \frac{R_f}{1-R_f} \frac{\varepsilon}{N^2}, \quad (1)$$

where  $N = \sqrt{\frac{g}{\rho} \frac{\partial \rho}{\partial z}}$  is a buoyancy frequency;  $g$  is a gravitational acceleration;  $\rho$  is density;  $R_f$  is flux Richardson number (the ratio of potential energy increase rate in the system to the rate of energy input spent on mixing) in acts of a stratified flux shear instability and breaking of wave disturbances. In [11–13], using various approaches, it was determined that  $R_f$  is constant for the phenomena under consideration. For application in calculations, several approximate  $R_f$  values were proposed (1/3 in [11] and 1/4 in [12]), as well as a value equal to 0.2 to estimate the common factor of the right-hand side of the second of formulas (1) – in [14–15]. Function  $\varepsilon$  is the energy dissipation rate in the act of breaking per unit area.

An approach to determining  $L$  is based on the structure analysis of the spectra of the first temperature fluctuation differences  $\frac{\Delta T'}{\Delta z}$  measured in the ocean [16]. It was previously determined that the effective vertical scale of natural turbulent patches corresponds to the vertical scale of a stable minimum in the vertical spectrum small-scale region of the first differences of fluctuations [17].

### **Turbulent mixing in the upper mixed layer**

Vertical turbulent exchange in the upper homogeneous layer of the sea is determined by a large number of natural factors. The main mechanisms for turbulence generation in this layer are instability of wave motions induced by surface waves, wave breaking, drift current velocity shear, convection, Langmuir circulation, and a number of others of lesser importance. Most models describing turbulent exchange in a layer adjacent to the atmosphere are limited to two or three of the mechanisms listed above. However, in recent years, researchers have been paying more and more attention to Langmuir circulations, suggesting their importance [18–21].

Convective mixing can be considered as a separate mechanism of vertical exchange, which, under appropriate conditions, can dominate over turbulent, but this type of mixing is not taken into account in the models considered in this paper.

Experimental data on the turbulent regime in the upper mixed layer (with neutral stratification) collected under various hydrometeorological conditions provided the verification of the most well-known models of turbulent exchange for this layer. The analyzed models included the model of the near-wall (logarithmic) layer, the possibility of which was proposed in [22], the models described in [23–25], as well as the multiscale model [26] developed at MHI Turbulence Department.

At this stage of research, a number of conclusions can be drawn from the comparative analysis of models and experiments.

1. At weak winds, the calculations based on none of the models provide satisfactory agreement with the experimental data: all the calculation results are obtained significantly lower.

2. The curve calculated according to Craig – Banner model [23], at moderate and strong winds, in many cases quite well falls on the experimental points but the roughness parameter  $z_0$  has to be changed within very large limits. In some cases, in order to achieve agreement between the results of calculations and experiment,  $z_0$  parameter must be many times higher than the wave height. It turns out a paradoxical situation when at a greater wave height the roughness parameter should be much less than in cases with small wave amplitude.

3. In addition to the current velocity shift as a turbulence generation mechanism, Benilov and Ly model [24] also takes into account the energy inflow from the surface waves. However, the proposed calculation method does not provide the desired effect – the calculated curve just slightly differs from the logarithmic curve (wall-bounded turbulence models for rough walls), and only in the uppermost part of the considered 1–2 m thick layer, below it these curves practically coincide. In most cases, these simulation results did not fit well with the measurement data <sup>2</sup>.

4. In the model of Kudryavtsev et al. [25], in most cases, the calculation results were less than the measurement data, which could be associated with the use of only one set of coefficients in the calculations, regardless of hydrometeorological conditions.

5. The results of calculations using the multiscale Chukharev model [26], as a rule, were in agreement with the experimental data, but in a number of cases a noticeable discrepancy was observed. The introduction of directly measured values of the wave characteristics and the current velocity profile into the model instead of parameterizations (especially in the upper 3 m layer) clearly improves the agreement between the results of calculations and experiments.

The results of various models verification indicate the need to include in the turbulent exchange models for the near-surface layer at least three main mechanisms of turbulence generation: drift current velocity shear, nonlinear effects of surface waves and their breaking. It is also important to note that the parameterizations of both the current velocity profile and the surface wave spectrum require further improvement, possibly taking into account the regional characteristics of the basin.

Thus, of the considered models, Chukharev model [26] turned out to be the most preferable for assessing the intensity of turbulent exchange in the near-surface layer of the sea. However, the model dependences used in this model (taken from the literature) of wave characteristics from wind velocity and current velocity profile do not always correspond to the actual values, which is reflected in the results.

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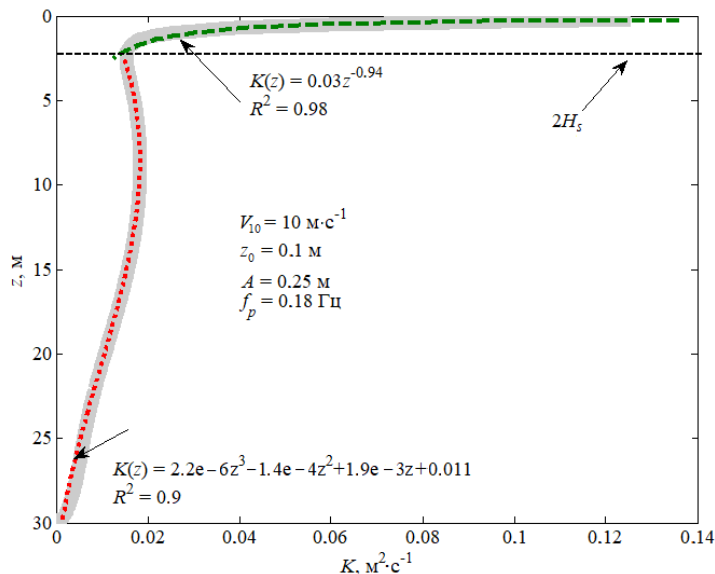
<sup>2</sup> Chukharev, A.M., 2014. [*Contribution of the Main Mechanisms of Turbulence Generation to Vertical Exchange in the Sea Stirring Layer*. Dr. Phys.-Math. Diss.]. Sevastopol: MHI RAS, 274 p.

An analysis of the relative contribution of various mechanisms to the overall generation of turbulence demonstrates that at different depths and under different hydrometeorological conditions, one or another mechanism can dominate. Thus, we again come to the conclusion that it is necessary to take into account all the main mechanisms, since neglecting any of them will distort the real picture.

The upper mixed layer turbulent exchange coefficient does not have a simple dependence on any one parameter and is a complex function of wind velocity, the degree of wave development, and others. In Fig. 3 an example of calculating  $K$  using a multiscale model with the input parameters shown in the figure are demonstrated. The calculation of  $K$  in the model is carried out through the turbulence energy and dissipation rate:

$$K = C_{\mu} \frac{E_t^2}{\varepsilon}, \quad C_{\mu} = 0.09.$$

The coefficient decrease in the uppermost layer occurs very rapidly, i.e. the direct effect of surface waves and their breaking is relatively shallow.  $K(z)$  dependence is well described here by the power function  $K(z) = az^n$ , where  $a = 0.005 \div 0.04$ ;  $n = -0.6 \dots -0.9$  at  $z_0 < z < 2H_s$ ;  $H_s$  is the height of significant waves. The values of numerical coefficients vary depending on the hydrometeorological conditions. For the lower layer ( $2H_s < z < 20H_s$ ), a polynomial dependence of the form  $K(z) = a_0 + a_1z + a_2z^2 + a_3z^3$  suits better, which is consistent with the  $KPP$  parameterizations usually used for the upper layer in global models. The turbulence generation by the velocity shear prevails here but the turbulence diffusion from the upper layer also affects.



**Fig. 3.** Dependence of the turbulent viscosity coefficient on the upper homogeneous layer depth: █ – calculation by the Chukharev model [24]; — — – coefficient approximation by a power function for the upper layer; · · · · – polynomial approximation for the lower layer. The figure shows the input parameter values of the model: wind speed at the height 10 m ( $V_{10}$ ), sea surface roughness parameter ( $z_0$ ), amplitude ( $A$ ) and frequency of the wave spectral peak ( $f_p$ )

Thus, the value of turbulent viscosity coefficient near the sea surface is a complex function of depth and significantly depends on the wind velocity, wave parameters, and other physical factors. In this case, the approximation of  $K$  below the wave layer is well described by a third-degree polynomial in  $z$ .

It is considered that the turbulence generated by surface waves and breakings is “faster”, but the scale of eddies is smaller than in shear turbulence, where they can reach the dimensions comparable to the mixed layer thickness.

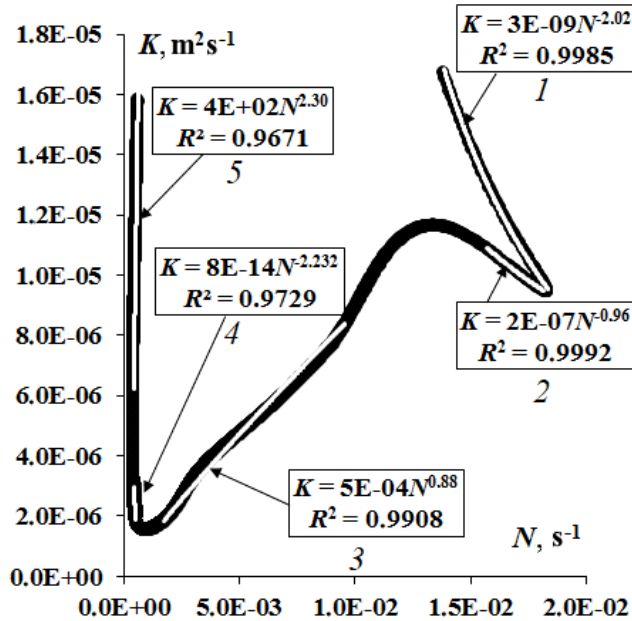
### **Turbulent exchange in the Black Sea stratified layers**

The main structural characteristic of layers in natural basins is well known – it is the presence of vertical density gradients that prevent the development of large-scale turbulence, but at the same time create favorable conditions for the development of internal waves (the shear instability formation dominant mechanism). In this case, the value of local Richardson number at the boundary of interlayers moving with multidirectional velocity becomes less than the critical one, which leads to the occurrence of small-scale turbulence, also due to the overturning of internal waves with the formation of turbulent patches. Inside such patches, turbulence is developed and, as a consequence, leads to vertical diffusion. The paper also discusses other mechanisms that generate turbulence in various layers of the Black Sea basin.

The purpose of this part of work is to identify the hydrophysical mechanisms of the layers formation with different density gradients observed in the Black Sea general stratified layer. As a solution to this problem, the construction of a 1.5D-model of vertical turbulent exchange for the studied stratified sea layer is presented. It includes the role of turbulent diffusion and vertical advection in different ways for various depth ranges. Based on this model, it is possible to identify differences in the vertical structure formation mechanisms for each layer. The first such model of vertical exchange for the Black Sea, including vertical advection and vertical turbulent diffusion, was constructed in [27, 28]. Later, some changes were made to the model [27] based on the results of the monograph [29]: the depth-averaged area of the studied basin and the mean temperature and salinity profiles in the upper stratified layer were refined. In this case, the method for calculating the depth distribution of the vertical diffusion coefficient for a long period from [27, 28] was used. Note that, in contrast to the previously mentioned works, with this approach, the sought functions are already calculated within 50–1750 m depth range [15].

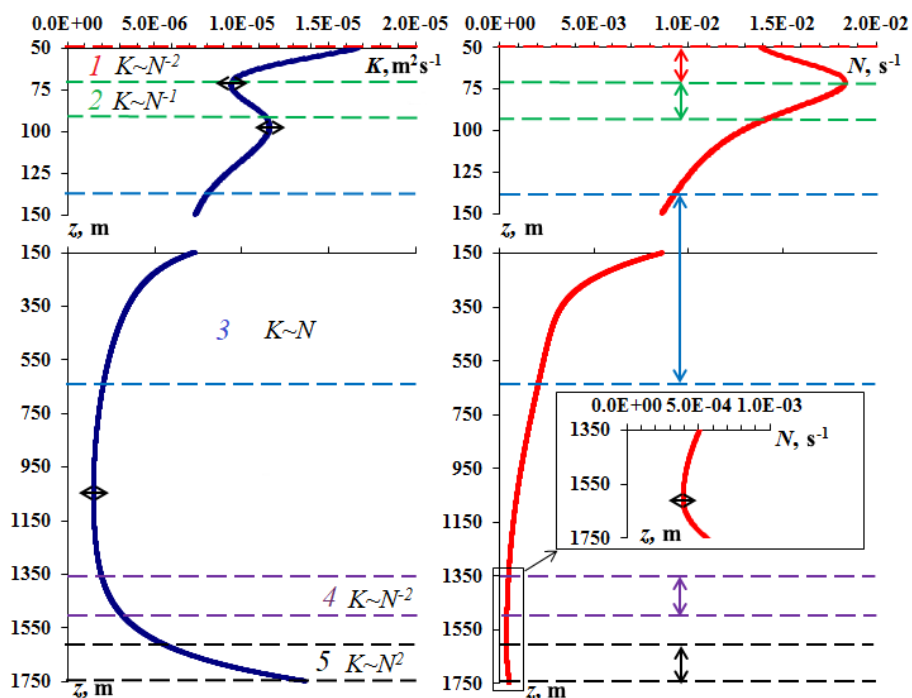
In the last decade, using the mentioned models and based on rich experimental data, we have obtained a number of important results characterizing the features of vertical turbulent diffusion in the Black Sea upper stratified layers [30–33]. Experimental data collected during the expeditions at R/V Professor Vodyanitsky from 2016 to 2018 provided, on the basis of a theoretical model [34], the development of ideas about the regularities of vertical turbulent diffusion in the main pycnocline.





**Fig. 4.** Dependence of the vertical turbulent diffusion coefficient  $K$  on the buoyancy frequency  $N$  in the stratified layers 1–5 at 50–1750 m depths

As indicated, the buoyancy frequency  $N$  (Väisälä – Brunt frequency) is used as the main parameter characterizing the density stability. Determination of the vertical turbulent exchange coefficient dependence  $K(N)$  is the main task of the described analysis. Calculation results comparison by semi-empirical and theoretical models revealed the fact that  $K(N)$  coefficient dependence is best approximated by a power function, the exponent of which is determined by the nature of internal waves in a particular medium [15]. A visual representation of  $K$  on  $N$  dependence variability is given in Fig. 4 and 5. The first of them shows (black line) the averaged dependence of the vertical diffusion coefficient  $K(N)$  within the framework of 1.5D-model. White lines indicate the approximating power-law dependences we have determined for various stratified layers. In the intervals between the calculated dependences  $K \sim N^\alpha$ , there are black curves connecting two different selected dependences  $K(N)$ . It should be noted that such a transition gap is practically absent between the dependences for layers 1 and 2 with negative  $\alpha$  powers.  $K(z)$  dependences in the layers 1–5 we have selected within the framework of 1.5D-model are shown in the left part of Fig. 5. On the right, the figure shows the averaged depth distribution of buoyancy frequency  $N$ . In each separate layer, the power dependence  $K(N)$  is indicated (Fig. 4) with a close integer degree  $\alpha$  at  $N$ . The following research results are presented in the table.



**Fig. 5.** Model distribution of the vertical turbulent diffusion coefficient  $K$  in the Black Sea stratified layers 1–5 (on the left), and dependence of the buoyancy frequency on depth (on the right) based on the average multi-year data ( $\leftrightarrow$  – the extrema)

### Dependences $K(N)$ , $\text{m}^2 \cdot \text{s}^{-1}$ , in different models for the stratified area of the Black Sea

Layers (according to Fig. 5)	1.5D-model	Theoretical models	Semi-empirical models
1. Lower stirring layer	$3 \cdot 10^{-9} N^{-2.02}$		$2 \cdot 10^{-8} N^{-1.8}$ [33] $2 \cdot 10^{-8} N^{-2}$ [9, 10]
2. Upper layer of the main pycnocline	$2 \cdot 10^{-7} N^{-0.96}$	$5.6 \cdot 10^{-5} N^{-1}$ [15]	
3. Lower layer of the main pycnocline	$5 \cdot 10^{-4} N^{0.88}$	$1.6 \cdot 10^{-2} N$ [15]	
4. The layer affected by a geothermal flow from the inclined bottom	$8 \cdot 10^{-14} N^{-2.2}$		$2 \cdot 10^{-14} N^{-2}$ [35]
5. Stratified boundary of the bottom layer	$4 \cdot 10^2 N^{2.3}$		

This table compares the dependences of vertical turbulent energy coefficient  $K$  on the buoyancy frequency  $N$  in various stratified layers of the Black Sea, obtained using 1.5D-model, as well as theoretical and semi-empirical models in earlier studies. Below we discuss the analysis results of five identified stratified layers of the basin under study.

The lower stirring layer 1 (cold intermediate) is located close to the upper stirring layer, which serves as a source of various disturbances for it. With a constantly maintained stratification in the layer under consideration, sources of disturbances from above form internal waves in it, which undergo breakings and maintain turbulent exchange. In addition, quasi-inertial internal waves (quasihorizontal stratified unstable currents) also contribute to the maintenance of vertical turbulent exchange in this layer due to local wave breaking and the formation of turbulent patches [34, 36]. In our work [33], the power function  $K(N)$  in the indicated layer was calculated based on the data analysis of high-resolution measuring complex “Sigma-1” (the probing version). As a result, it was revealed that the power-law dependences of the buoyancy frequency in this layer, calculated from the measurement data ( $N^{-1.8}$ ) and from the modeling results using 1.5D-model ( $N^{-2.02}$ ) (see table), are close. For the rest of the studied layers, there are no measurement data due to the fact that the working immersion depth of the measuring probe complex is limited to 300 m. At the same time, there is another semi-empirical method that confirms  $K(N)$  dependence in the specified layer:  $\varepsilon \propto L^2 N^3$ ,  $K \propto \varepsilon \cdot N^{-2}(1)$ . Hence, using the relation [9, 10], from formulas (1) we obtain the required dependence  $K \cong 2 \cdot 10^{-8} N^{-2} \text{ m}^2 \cdot \text{s}^{-1}$ . This value is given in the table together with the results of calculations for 1.5D and semi-empirical models in the layer under study.

The next two stratified layers (2 and 3) constitute the main pycnocline of the studied basin. As was determined in [15, 34], vertical turbulent exchange is formed here due to shear instability of ray quasi-inertial internal waves (quasihorizontal currents) in a stratified fluid. In contrast to the lower, weakly stratified layer, in the upper, strongly stratified layer, the characteristic scales of waves transferring their energy to the turbulence (as shown) also depend on the derivative of the buoyancy frequency function  $N(z)$ . As a result, the following results were obtained for the upper and lower layers:

$$\begin{aligned} \varepsilon \propto N^3 \left| \frac{\partial N}{\partial z} \right|^{-1}, \quad K \propto N \left| \frac{\partial N}{\partial z} \right|^{-1} \quad (\text{layer 2}); \\ \varepsilon \propto N^3, \quad K \propto N \quad (\text{layer 3}). \end{aligned}$$

The variation in depth dependence is exclusively due to the significant difference in the values of the buoyancy frequency  $N$  (vertical density gradients) in the upper and lower parts of the main pycnocline. More precisely, the pycnocline model is a structure of two layers described by two different power-law dependences  $K(N)$  and the function  $N(z)$  that unites them (see Fig. 4 and the Table). The power-law dependence  $N(z)$  in the main pycnocline (sum of layers) has the form  $N \propto z^{-1}$ . In this case, the relations for the upper layer 2 are constructed using the real dependence  $N(z)$ :  $\varepsilon \propto N$ ,  $K \propto N^{-1}$ . The obtained result demonstrated

close power-law dependences of buoyancy frequencies  $N^n$  with the 1.5D model for both layers (see Fig. 5, Table).

In layer 4, the dependence of vertical turbulent diffusion coefficient  $K$  on the buoyancy frequency  $N$ , calculated within the framework of 1.5D-model, has the form  $K \propto N^{-2.2}$  (see Figs. 4 and 5). The probable hydrophysical mechanism of turbulent exchange in this deep-water layer is presented in [35]. In this paper, a model of vertical exchange has been developed due to geothermal heat flux from the inclined bottom, causing intrusive layering in the lower stratified layer of the Black Sea. The basin shape in the model is assumed to be conical, locally varying in depth, and the circle radius is determined in accordance with the change in the natural local area of the basin. The general expression for the vertical diffusion coefficient is

$$K = \varepsilon_p N^{-2},$$

where  $\varepsilon_p$  is an energy dissipation rate (with regard to coefficient from formula (1)), here the bottom topography, bottom geothermal flux and the efficiency of heat engine are taken into account. As a result, the model dependence of vertical turbulent exchange coefficient has the form  $K \sim 2 \cdot 10^{-14} N^{-2} \text{ m}^2 \cdot \text{s}^{-1}$  (see the Table). It should be noted that, in contrast to the model [35], which was constructed in the depth range from about 800 m to the upper boundary of the near-bottom mixed layer, the dependence in the model used in this experimental approach ( $K \propto N^{-2}$ ) actually manifested itself only in a “layer” of about 200 m thickness (see Fig. 4 and 5). This indicates that in the studied depth range we observe the layers in which a joint formation mechanism of vertical turbulent exchange, which manifests itself as a “gap” in the depth between the layers where the approximating dependence is not determined. For example, the same “gap” is also noted between layers 3 and 4. In this case, it should be noted that the mechanism under consideration manifests itself in the upper part of layer 4 more noticeably than was observed in the main pycnocline (aggregate of layers 2–3). In the upper layer adjacent to layer 3, the function  $K(N)$  begins to increase (in contrast to layer 3, in which it decreased) from a depth of approximately 1000 m (the point is marked as an extremum in Fig. 5), but with a different dependence on  $N$ .

The last of the considered stratified layers – layer 5 – is located at 1600–1750 m depths, below the stratified layer 4, and precisely adjoins the bottom homogeneous layer, not shown in the figures. The vertical structure of the stratification of two layers, 4 and 5, is shown in Fig. 5. In [36], the stationary state of the bottom layer is considered. The heat, continuously coming from the bottom, maintains the stationarity of the homogeneous layer and, together with the dissolved salt constantly getting from the flowing lower Bosphorus current, penetrates upward through a thin stratified layer 5 adjacent from above. The bottom layer can be represented as a heat engine, in which the only source of energy for maintaining the vertical exchange in the system is the bottom heat flux [37]. For such a system, it is fair to use the dependences of  $\varepsilon$  and  $K$  on the buoyancy frequency  $N$  in accordance with relations (1). However, since currently there are no developed exchange models,  $L(N)$  dependence in layer 5 can be found using the vertical

turbulent diffusion coefficient from 1.5D-model (Fig. 4). Then the desired dependence of turbulent patches scale takes the form

$$L \propto N^{1/3}.$$

It should be noted that the expression  $L(N)$  is applicable to this situation. Indeed, the lower boundary of layer 5 is linked to the homogeneous layer upper boundary where the gradients should vanish. Consequently, on the lower boundary  $N = 0$ , and the solution obtained here  $L_b = 0$ ,  $K_b = 0$  satisfies this requirement. In this case, the presence of “turbulent patches” in the upper layers, naturally, does not turn the exchange at the boundaries to zero:

$$(I)L \propto N^{-1}; \quad (II)L \propto N^{-2/3}; \quad (III)L \propto N^0; \quad (IV)L \propto N^{-1}.$$

### Conclusions

Summarizing the foregoing, we can say that the performed experimental and theoretical studies make it possible to make practical estimates of vertical mixing intensity in different layers of the Black Sea.

For the upper mixed layer, various models of turbulent exchange were verified and the limits of their applicability were determined for different hydrometeorological conditions. The dependence of the exchange coefficient on depth is determined by the effect of dominant turbulence source at a given horizon.

For stratified sea layers, theoretical and semi-empirical relations for the coefficient of vertical turbulent exchange depending on the buoyancy frequency are obtained. It should be noted that the results comparison of calculating the dependence  $K(N)$ , which has the form  $K \cong AN^\alpha$ , according to 1.5D-model and hydrophysical models, shows a significant difference in the factor  $A$  values. Differences in the values of the coefficients (factors) may be due to the peculiarities of the Black Sea bottom topography and unevenness in the exchange processes distribution of various origins. So, for layer 1, the factor differs by an order of magnitude, which is most likely due to the close layer location to the upper 50-m mixed layer, while for layers 2 and 3 the discrepancy in the coefficients increases by two orders of magnitude due to the limited region of quasi-horizontal internal waves' generation.

Based on the analysis of 1.5D-model, the dependence of vertical turbulent diffusion coefficient  $K$  on the buoyancy frequency  $N$  in five identified stratified layers of the Black Sea basin at depths from 50 to 1750 m was revealed. Based on the analysis of a number of previously constructed semi-empirical vertical exchange models, various physical mechanisms for maintaining vertical turbulent exchange in different layers of the basin have been identified. A comparison of the calculation results using 1.5D-model and semi-empirical models demonstrates that the powers of  $\alpha$  at  $N$  in the dependence  $K \sim N^\alpha$  have similar values. When analyzing the results, the layers thickness was limited by the condition of power constancy in the layer, i.e. most layers did not touch each other. As it was found out, this is explained by the fact that different physical mechanisms of vertical exchange often act simultaneously in one stratified layer.

In order to clarify the causes for the difference between the model and experimental coefficients in  $K = AN^\alpha$  dependences, it is necessary to increase the number of measurements with the determination of local and seasonal differences in  $A$  coefficient and  $\alpha$  power.

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*About the authors:*

**Anatoliy S. Samodurov**, Chief Research Associate, Head of Turbulence Department, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., Sevastopol, 299011, Russian Federation), Dr. Sci. (Phys.-Math.), **ORCID ID: 0000-0002-9910-5325**, [anatol\\_samodurov@mail.ru](mailto:anatol_samodurov@mail.ru)

**Aleksandr M. Chukharev**, Leading Research Associate, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., Sevastopol, 299011, Russian Federation), Dr. Sci. (Phys.-Math.), **ORCID ID: 0000-0003-1078-6425**, [alexchukh@mail.ru](mailto:alexchukh@mail.ru)

**Dmitry A. Kazakov**, Junior Research Associate, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., Sevastopol, 299011, Russian Federation), Postgraduate, **ORCID ID: 0000-0001-5083-4968**, [engineer.dk@mail.ru](mailto:engineer.dk@mail.ru)

*Contribution of the authors:*

**Anatoly S. Samodurov** – research task setting, mathematical model development, review of literature on the vertical turbulent exchange models for stratified layers of the Ocean, choice of data analysis methods, formulation and analysis of the results obtained in the study, their interpretation, discussion of the work results, formulation of conclusions, article writing

**Alexander M. Chucharev** – research task setting, mathematical model development, review of literature on the vertical turbulent exchange models for near-surface quasi-homogeneous layer of the Ocean, software development and calculations, plotting, qualitative analysis of the research results and their description, formulation of conclusions, article writing

**Dmitrii A. Kazakov** – data processing and analysis, verification of vertical turbulent exchange models in stratified ocean layers, calculation results analysis and their interpretation, plotting, discussion of the work results, formulation of conclusions, article writing and editing

*All the authors have read and approved the final manuscript.*

*The authors declare that they have no conflict of interest.*