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Original article

Ratio between Trough and Crest of Surface Waves in the Coastal Zone of the Black Sea

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Abstract
Purpose. The work is aimed at analyzing variability of the ratio between trough and crest of the sea surface waves, as well as the relationship of this ratio with the skewness of sea surface elevations.
Methods and results. The analysis is based on the wave measurements performed from the stationary oceanographic platform located near the Southern Coast of Crimea in the Black Sea. The depth at the place where the platform is installed is about 30 m. The analyzed data array totals 17,083 twenty-minute measurement sessions. The freak waves were identified by the abnormality index *AI* equal to the ratio between the maximum wave height per session and the significant wave height. The freak waves with index $AI > 2$ were observed in 562 measurement sessions. This corresponds to a probability of their occurrence equal to 3.3%. The *AI* values range from 1.16 to 2.79. The ratio between the trough *Th* of the highest wave and its crest *Cr* is in the range $0.37 < Th/Cr < 1.47$, at that the average value is 0.79.
Conclusions. Statistical characteristics of the waves revealed in the presence of freak waves differ noticeably from those obtained at $AI < 2$. In the situations when $AI < 2$, the probability of an event when the trough *Th* of the highest wave exceeds its crest *Cr* is 10.9%. The event with $Th/Cr > 1$ does not occur if $AI < 1.4$. When there are waves satisfying condition $AI > 2$, the probability of an event $Th/Cr > 1$ is 19.4%. It is shown that condition $Th/Cr > 1$ is not necessary for arising of a negative skewness of sea surface elevations. The probability of skewness large deviations from a zero value both towards positive values and towards negative ones, is higher at $AI > 2$ than at $AI < 2$. The statistical relationship between the skewness and the Th/Cr ratio is observed only for freak waves.

Keywords: sea surface, freak wave, abnormality index, skewness, Black Sea

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Introduction

In the classical representation, sea surface waves have a trochoidal form with a pointed crest and a flat trough. Waves of this form correspond to the positive skewness of surface elevations. Work [1] published in 1963 shows that in the case when sea surface structure is formed by free undamped waves, the skewness is always positive. Currently, several types of models are applied to describe the distribution of weakly nonlinear wave field: those constructed on the basis of the Stokes expansion [2, 3] and those in which the cumulants of sea surface elevations are described by multidimensional integrals of wave spectra [4, 5]. The skewness determined within the framework of these models is always positive as well.

At the same time, numerous measurements carried out in the various World Ocean regions indicate the situations when the skewness of sea surface elevations is negative [6–10]. A possible cause why weakly nonlinear random wave models do not describe these situations lies in the fact that they are constructed for a statistically homogeneous field in which nonlinearity manifests itself in the form of minor corrections [3]. A real wave field includes mechanisms leading to local effects that cannot be considered weakly nonlinear ones. In particular, these are freak waves [11] the presence of which shows significant deviations of skewness and kurtosis from zero values. Skewness can reach a value of –0.4, kurtosis can exceed one [7, 8].

Several forms of freak waves [12, 13] with different ratios of crest and trough are distinguished, including the waves in which trough is greater than crest. The presence of such waves should lead to a shift in the skewness of sea surface elevations towards negative values.

This work is purposed at analyzing the variability of the ratio between trough and crest of the surface waves, as well as the relationship of this ratio with the skewness of sea surface elevations.

Equipment and measurement conditions. Studies of the field of sea surface waves were carried out at a stationary oceanographic platform located in the Black Sea coastal part off the Southern Coast of Crimea. Measuring equipment, as well as the features of carrying out measurements from a stationary oceanographic platform, are described in [9, 14–16]. The sea depth at the place where the platform was located is ~ 30 m. For typical Black Sea waves, the indicated depth corresponds to deepwater conditions.

Measurements were carried out from May 2018 to January 2019 with several short breaks. For statistical analysis, continuous measurements were divided into sessions lasting 20 minutes each, from which the characteristics of the waves were determined.

Trough/crest ratio. The abnormality index (AI) is applied to identify freak waves [17–19]

$$AI = H_{\max} / H_S ,$$

where $H_{\max}$ is maximum wave height during the measurement session; H_S is significant wave height equal to 1/3 of the average height of the highest waves. It is generally accepted that the waves with a height that exceeds significant wave height more than twice are freak waves, i.e. those waves for which $AI > 2$.

Another less common criterion for identifying freak waves is based on the ratio [20]

$$CI = Cr / H_S ,$$

where *Cr* is height of the maximum wave crest. Waves with *CI* exceeding the critical value, which is set equal to 1.2, 1.25 or 1.3, are considered freak waves. The relationship between *AI* and *CI* parameters was analyzed in [19]. It was shown that the application of *CI* criterion resulted in an underestimation of the number of situations when freak waves are observed, compared to the estimate obtained using the criterion $AI > 2$. Discrepancy is due to existence of freak waves crest of which is less than trough.

The data array analyzed in this work consisted of 17,083 measurement sessions. Freak waves (according to criterion $AI > 2$) were recorded in 562 sessions. This corresponds to the probability of their occurrence equal to 3.3%. For comparison, we point out that during the measurements off the western Black Sea coast of Turkey (12.5 m depth), the probability of freak waves occurrence was 2.6% [8].

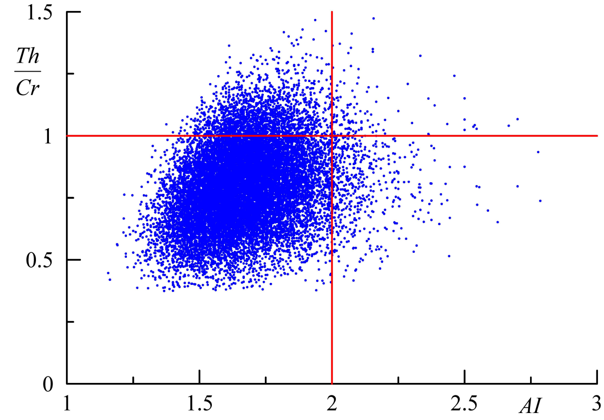
The data array was obtained at wind speeds W_{10} from calm (conditionally 0 m/s) to 26 m/s with an average speed of 5.6 m/s over the entire measurement period. Here, the wind speed W_{10} is normalized to a horizon of 10 m. Significant wave heights during this period varied in the range of 0.04–2.27 m with an average value of 0.55 m/s. Variation of W_{10} and H_S occurred in almost the same ranges in situations when freak waves were observed: W_{10} varied in the range of 0–21.5 m/s with an average value of 5.4 m/s; H_S – in the range of 0.06–2.1 m with an average value of 0.52 m.

Three forms of freak waves can be distinguished according to the classification proposed in [13]. The form in which crest *Cr* is one and a half times greater than trough *Th* is positive. The form when *Th/Cr* > 1.5 is negative. The third form is intermediate. The probability of occurrence of these three forms was 63, 17.5 and 19.5%, respectively. The second form waves were not observed in our measurements; the maximum *Th/Cr* ratio is 1.47. It can be assumed that large *Th/Cr* values are due to the fact that wave measurement data were analyzed in [13] at shallow depths (2.7 m) where high nonlinearity caused by the interaction of surface waves with the bottom takes place.

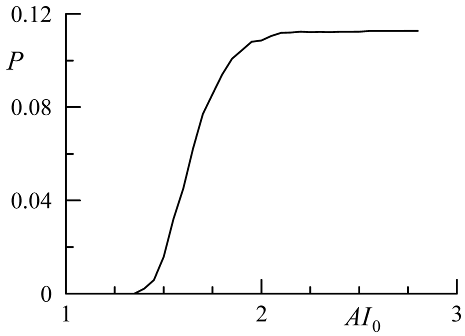
According to measurements carried out from the stationary oceanographic platform, as well as according to previous studies, freak waves with *Th*/*Cr* < 1 predominate. *Th/Cr* ratio ranges from 0.37 to 1.47, with an average value of 0.79. In situations when *AI* > 2, the probability of occurrence of waves with *Th/Cr* < 1 was 19.4%. The probability that *Th/Cr* > 1 (calculated for the conditions when freak waves are not observed) was 10.9%, average probability for the ensemble of all situations is 11.3%. Fig. 1 shows *Th/Cr* dependence on the abnormality index.

Fig. 2 shows conditional probability of event $P\left(Th/Cr > 1 \middle| AI < AI_0\right)$, at which $Th/Cr > 1$ if the abnormality index does not exceed a certain critical value AI_0. It can be seen that the event $Th/Cr > 1$ does not occur if $AI < AI_0 = 1.4$. The conditional probability was constructed for a wave measurement data set in which *AI* parameter varied within 1.16 and 2.79.

Conditional probability $P\left(Th/Cr > 1 \middle| AI < AI_0\right)$ increases rapidly in *AI* < 2 region and varies slightly when *AI* > 2. Based on Fig. 2, it can be assumed that the statistical characteristics of waves in the presence of freak waves differ from the statistical characteristics in cases when they are not observed. We provide other evidence of the validity of this assumption below.



F i g. 1. Dependence of the ratio between trough and crest *Th*/*Cr* on the abnormality index *AI*



F i g. 2. Conditional probability $P(Th/Cr > 1 | AI < AI_0)$

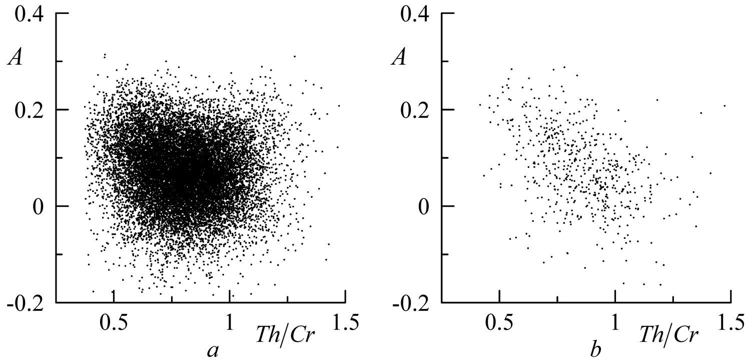
Skewness. One of the main criteria for the nonlinearity of the field of sea surface waves is the deviation of statistical moments from the values corresponding to the Gaussian distribution [10, 21]. Assuming that the mean value of sea surface elevation is zero, the skewness A of sea surface elevations can be defined as

$$A = \mu_3 / \mu_2^{1,5},$$

where $\mu_n = \langle \xi^n \rangle$ is statistical moment of order n; ξ is surface elevation; symbol $\langle \ldots \rangle$ means averaging.

It is natural to assume that the occurrence of negative skewness values is statistically related to the ratio Th/Cr. Let us consider two arrays of wave measurement data. The first array includes all data, the second – only the data obtained in the presence of freak waves. If the analysis is carried out for the entire range of *AI* variations, then the relationship between A and Th/Cr is not observed (Fig. 3, *a*) and correlation coefficient ρ between these parameters is equal to – 0.1.

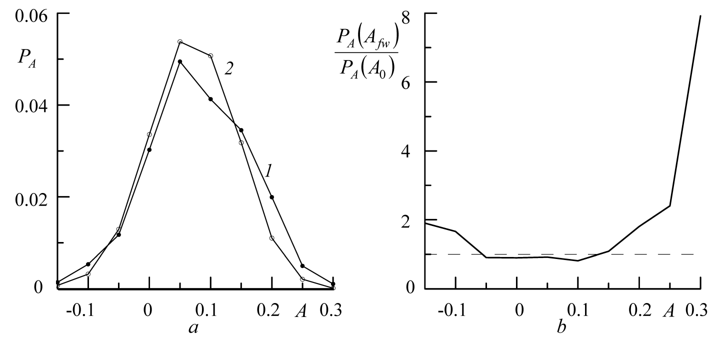
Statistical relationship between A and Th/Cr occurs only when AI > 2 (Fig. 3, *b*) and correlation coefficient is $\rho = -0.42$ for the second data array.



F i g. 3. Dependence of skewness A on the ratio between trough and crest Th/Cr: *a* – within the entire range of AI variation; *b* – at AI > 2

It is noteworthy that negative skewness values can occur when two conditions are simultaneously satisfied, Th/Cr < 1 and AI < 2.

Let us consider two groups of skewness estimates. Estimates A_{fw} were obtained when freak waves were observed (AI > 2), estimates A_0 – when freak waves were absent (AI < 2). Fig. 4 demonstrates the probability density functions of estimates A_{fw} and A_0 (we denote them as $P_A\left(A_{fw}\right)$ and $P_A\left(A_0\right)$, respectively), as well as the ratio $P_A\left(A_{fw}\right)/P_A\left(A_0\right)$. Probability density functions were calculated as histograms normalized by the sample length and the width of interval within which the skewness estimates fell. The probability of large skewness deviations from zero value at AI > 2 is higher than at AI < 2. This applies to deviations towards both positive and negative values.



F i g. 4. Changes in the distribution of skewness of surface elevations in the presence of abnormal waves: *a* – probability density functions of skewness, curve *1* – $P_A\left(A_{fw}\right)$, curve *2* – $P_A\left(A_0\right)$; *b* – ratio $P_A\left(A_{fw}\right)/P_A\left(A_0\right)$

Average values calculated for two groups of skewness assessments are close: $\langle A_{fw} \rangle = 0.079$ and $\langle A_0 \rangle = 0.072$.

Conclusion. The probability of occurrence of freak waves (according to criterion *AI* > 2) in the Black Sea off the Southern Coast of Crimea when the deep water condition is satisfied is 3.3%. It is shown that statistical characteristics of the waves revealed in the presence of freak waves differ noticeably from those obtained at *AI* < 2. It was found that the probability of event when trough depth *Th* of the highest wave exceeds the height of its crest *Cr* is 10.9%. The probability of *Th/Cr* > 1 event was 19.4% with the occurrence of waves with abnormality index *AI* > 2. The event *Th/Cr* > 1 does not occur if *AI* < 1.4.

The probability of large skewness deviations from zero towards both positive and negative values for *AI* > 2 is higher than for *AI* < 2. The condition *Th/Cr* > 1 is not necessary for arising of negative skewness of sea surface elevations. Negative skewness values can be observed when two conditions are satisfied simultaneously: *Th/Cr* < 1 and *AI* < 2. Statistical relationship between skewness and *Th/Cr* ratio is observed only when *AI* > 2.

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Contribution of the co-authors:

Aleksandr S. Zapevalov – formulation of goals and objectives of the study; review of literature on the research problem; analysis of measurement data; writing of the article text

Anton V. Garmashov – processing and description of measurement results; analysis and synthesis of research results; statistical analysis; addition to the article text

The authors have read and approved the final manuscript.
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