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Original article

Numerical Study of Resonant Oscillations of Water Level in the Sea of Azov under the Impact of Weak External Forces

A. L. Chikin

Federal Research Center the Southern Scientific Center, Russian Academy of Sciences, Rostov-on-Don, Russian Federation
✉ *chikin1956@gmail.com*

Abstract
Purpose. The study is aimed at determining the frequencies of resonant oscillations of water surface level in the Sea of Azov.
Methods and Results. The computational experiments were performed using a mathematical model based on a system of equations for long waves in a homogeneous fluid. The wind of variable direction or strength and atmospheric pressure oscillations were considered to be an external force. The problem was solved by the finite-difference methods using the implicit difference schemes. The values of period lengths at which the resonant effect arises were obtained. The oscillation frequency values were determined by calculating the total energy of oscillations as a sum of potential and kinetic energy. Agreement between the values of resonant frequency (or periods) of sea level oscillations under conditions of variable wind load and changing atmospheric impact has been revealed. It has been established that stable resonant oscillations emerge after four to seven periods of external periodic forcing, after which the oscillatory motion stabilizes.
Conclusions. The applied mathematical model permits to conduct computational experiments for studying the process of formation of resonant oscillations of the Azov Sea water level. The results of investigation describe adequately the periodic external influence on the water surface of the Sea of Azov. The calculated resonance frequencies are consistent with the results obtained by other authors.

Keywords: shallow water equations, resonance, resonant oscillations, seiche, computational experiment

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Introduction

Oscillatory motions of water in enclosed or semi-enclosed water bodies can arise under relatively weak physical forcing on the water surface if this forcing resonates with the natural oscillations of the water body. The Sea of Azov belongs to such water bodies. Oscillations generated by setting the entire water mass of the Sea of Azov into motion affect its hydrological regime. In many studies, such resonant oscillations are identified as seiches and modeled by applying a disturbing force to the water surface followed by an analysis of free surface oscillations after the cessation of the forcing. Disturbances can be specified as either wind forcing or surface oscillations caused by atmospheric pressure changes. To identify seiches, forced oscillations are calculated to identify resonance with the natural oscillations of the water body.

Seiches can occur when external forcing, even of small amplitude, coincides with the natural frequencies of a water body's oscillations, posing significant risks to anthropogenic infrastructure and ecosystems. The work [1] analyzes the long-term dynamics and recurrence of dangerous and adverse level oscillations in the Sea of Azov, leading to catastrophic rises in the water level, shoreline destruction, and the flooding of the Dolzhanskaya, Yeyskaya, Chumburskaya, and Ochakovskaya spits, and damage to coastal structures.

Using mathematical modeling, the work [2] investigates the influence of inhomogeneous moving atmospheric pressure fields on currents, free and forced level oscillations in the Sea of Azov caused by constant wind action and barometric disturbances. Free and forced oscillations in the Sea of Azov were studied using mathematical modeling with the three-dimensional sigma-coordinate POM (Princeton Ocean Model).

In [3], the water level oscillations (surge) near the Kerch Strait in the Sea of Azov are specified as the forcing. The resonant frequency is determined by the maximum total energy value observed while varying the frequency of the exciting oscillations. Total energy is defined as the sum of kinetic and potential energy.

In [4], the water level and current velocity oscillations are specified for the Balaklava Bay. Initially, the fluid is at rest. Then, a periodic disturbance begins acting on the open boundary of the basin. The objective is to determine the periods of the basin's natural oscillations and to study the structure of the level and current fields at the obtained period values.

The objective of the work [5] is to find an analytical solution to the seiche problem within the linear approximation for a rectangular basin of constant depth, which allows one to calculate the periods of seiche oscillation modes, free surface deviations, and wave current velocities. Using the obtained solution, seiche and seiche-like oscillations are then studied. The solution is sought in the form of time-periodic functions. Calculations were carried out for a rectangular basin 450 km long, 250 km wide, and 10 m deep, serving as an approximate model of the Sea of Azov, given its similar dimensions.

In [6], free sea level oscillations were modeled hydrodynamically, taking into account body forces, such as the Coriolis force and bottom friction. A nonlinear system of shallow water equations was used as the governing system. Above the sea, with initial values of level and currents set to zero, a uniform, constant wind blowing from different directions was specified. After 10 hours, the wind was switched off, and the free level oscillations were examined.

A similar approach to studying free water level oscillations in the Sea of Azov arising after the cessation of constant wind action is presented in [7]. This paper uses a sigma-coordinate model to determine the characteristics of seiche-like oscillations.

In [8], the speed and time of movement of the barometric field are determined based on the assumption that waves are generated with maximum amplitude. This occurs when the period of the forcing is similar to the period of the basin's natural oscillations. The time it takes the front to traverse the entire sea area is set equal to the time it takes for the Sea of Azov level to rise to its maximum, which occurs during half the period of the fundamental mode of free oscillations.

Based on the analysis of numerical modeling results in [9], seiche-like level oscillations in the Sea of Azov within the field of atmospheric pressure disturbances were studied. In this case, the period of the atmospheric pressure disturbances equals the period of the fluid's natural oscillations in the basin.

This work aims to numerically investigate the occurrence of resonant oscillations of the Sea of Azov water surface resulting from weak periodic external forcing. This forcing could be a weak wind with variable direction or strength, or atmospheric pressure that changes with a specific period. Computational experiments were conducted using a mathematical model developed at the Southern Scientific Centre of the Russian Academy of Sciences [10].

Materials and methods

Calculations of level oscillations in the Sea of Azov are based on solving a system of equations for long waves in a homogeneous incompressible fluid subject to Coriolis force, while accounting for atmospheric pressure, as described in [11]:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\Omega v=-g\frac{\partial}{\partial x}\left(\zeta+\frac{P_a}{g\rho_0}\right)+\frac{\tau_{sx}}{H}-\frac{\tau_{bx}}{H},$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\Omega u=-g\frac{\partial}{\partial y}\left(\zeta+\frac{P_a}{g\rho_0}\right)+\frac{\tau_{sy}}{H}-\frac{\tau_{by}}{H},$$

$$\frac{\partial \zeta}{\partial t}+\frac{\partial (Hu)}{\partial x}+\frac{\partial (Hv)}{\partial y}=0\,,$$

where $H=h+\zeta$; $h=h(x,y)$ is the water body depth; $u=u(x,y,t)$, $v=v(x,y,t)$ are velocities; P_a is the atmospheric pressure; ρ_0 is the mean density of the seawater; τ_{sx},τ_{sy} are the projections of the wind friction force against the water surface onto the *OX* and *OY* axes; τ_{bx},τ_{by} are the projections of the fluid friction force against the bottom onto the *OX* and *OY* axes. A similar approach to modeling marine floods in the Don River delta is presented in [12]. The values of the friction forces depend on the wind speed $\overline{W}_{\mathrm{B}}=\{W_x;W_y\}$ and current velocity $\overline{W}_{\mathrm{T}}=\{u_s;v_s\}$ and are determined as follows [13]:

$$\overline{\tau}_s=\gamma\left|\overline{W}_{\mathrm{B}}\right|\overline{W}_{\mathrm{B}},\qquad \overline{\tau}_b=\beta\left|\overline{W}_{\mathrm{T}}\right|\overline{W}_{\mathrm{T}}\,,$$

where $\left|\overline{W}_{\mathrm{B}}\right|=\sqrt{W_x^2+W_y^2}$, $\left|\overline{W}_{\mathrm{T}}\right|=\sqrt{u^2+v^2}$; β is the bottom friction coefficient; γ is the wind friction coefficient against the free water surface.

Slip conditions are specified at the solid boundary $\partial\Omega_b$:

$$\mathbf{V}_n\big|_{\partial\Omega_b}=0,\qquad \left.\frac{\partial \mathbf{V}_\tau}{\partial \overline{n}}\right|_{\partial\Omega_b}=0\,.$$

The problem was solved using finite-difference methods on a uniform grid with implicit difference schemes. The convective terms of the momentum equation were approximated using upwind differences. The water level difference was determined using the corresponding finite-difference analog. The grid spacing was Δx = 660 m and Δy = 685 m with 524 × 354 nodes. After indexing the cells in the computational domain, the number of unknowns for each variable was approximately 83,000. Although an analytical assessment of the permissible time step in the difference schemes was not performed, numerical calculations showed computational stability at Δt < 120 s. The program is written in FORTRAN; numerical implementation of the model was carried out on high-performance computing systems in an MPI environment using the Aztec package of parallel subroutines.

The resonant frequency of free and forced oscillations was determined by computing the time-averaged total energy as the sum of kinetic energy E_k and potential energy E_p. The total energy calculation followed the methodology from [3]:

$$E = E_k + E_p = \frac{\rho}{2T}\left(\iint\limits_{TS}(H+\zeta)\left(u^2+v^2\right)dSdt + g\iint\limits_{TS}\zeta^2 dSdt\right).$$

Here ρ is the density of seawater (assumed constant in the model and equal to 1000 kg/m³); T is the oscillation period; S is the computational domain area. When constructing graphs, the natural logarithm of the total energy $\ln(E)$ was considered.

Results and discussion

The range of variation of the forced oscillation period was studied over a time span of 2 to 30 hours. Resonance was observed at frequencies with a local maximum in total energy. The main scenarios considered for the occurrence of resonant water level oscillations in the Sea of Azov are as follows:

– wind direction changes from NE to SW at a constant speed of 2 m/s;

– an easterly wind action with a speed of 2 m/s alternates with calm;

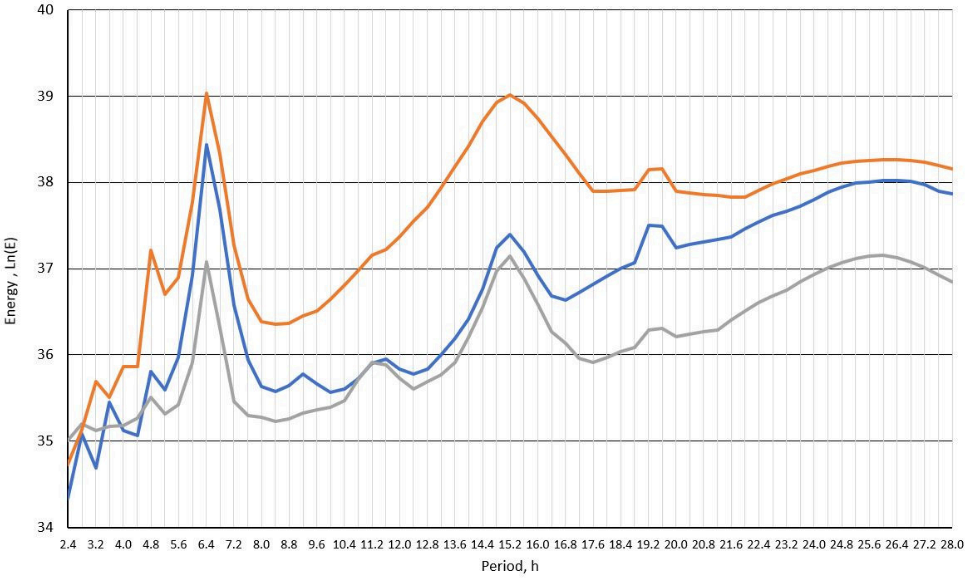
– atmospheric pressure at the eastern and western boundaries of the Sea of Azov fluctuates alternately from 750 to 770 mm Hg.

In addition to the main scenarios, the wind directions NW–SE and N–S were also considered.

The period of forced oscillations changed every 0.2 hours (change in wind load or pressure). To obtain a more stable resonance pattern, the external forcing was modified 15 times for each period.

The most prominent local maximum energy was observed several times for period values of 6.4, 15.2, 19.2, and 26 hours (Fig. 1). This occurred with NE–SW winds, the alternation of an easterly wind and calm periods, and for atmospheric pressure oscillations. Furthermore, several smaller energy spikes were identified. These frequencies were observed for all wind directions and corresponded to both longitudinal and transverse forcing of the sea surface. The only difference was in the magnitude of the total energy.

The situation where a 2 m/s wind changed direction from SE to NW and alternated between easterly direction and calm was considered. The most pronounced local energy maxima were observed at periods of 6.4, 15.2, and 26 hours, consistent with results obtained by other authors. For instance, work [14] shows that three oscillation modes can be distinguished in the level oscillations of the Sea of Azov: 23, 14.5, and 6–8 hours. Higher oscillation modes are less significant and not characteristic of the entire sea. Oscillations with a period of 23.7 hours are mentioned in [3], but the author attributes them to a two-node seiche.

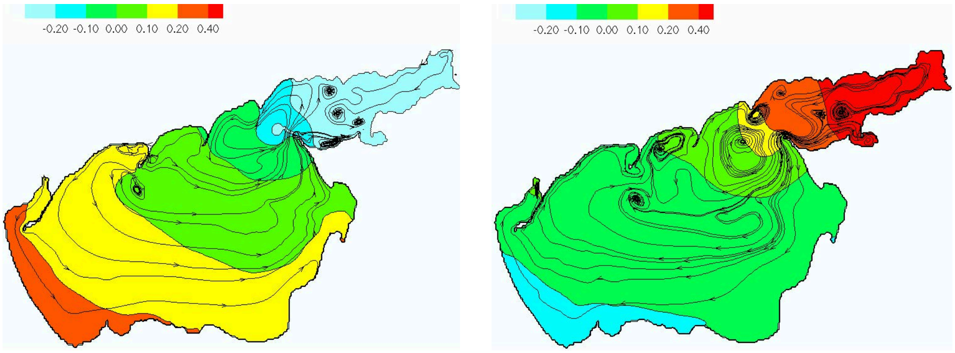


F i g. 1. Dependence of the energy logarithm on wind impact frequency at: changing the direction of constant wind (speed 2 m/s) from NE to SW (blue curve) (*a*), alternating action of east wind (speed 2 m/s) and calm (gray curve) (*b*), and fluctuations in atmospheric pressure at the eastern and western boundaries of the Azov Sea in the range 750–770 mm Hg (orange curve) (*c*)

The figures below illustrate the distribution of the sea surface elevation at the maximum total energy value.

With a wind oscillation period of 26 hours, water motion in the Sea of Azov exhibits a circulatory pattern, forming several circulation zones in both the Sea of Azov and Taganrog Bay. Fig. 2 shows the distribution of sea surface elevation and streamlines, displaying a maximum in Taganrog Bay (0.85 m) and a minimum near the Arabat Spit (−0.15 m), alternating with a minimum in Taganrog Bay (−0.74 m) and a maximum near the Arabat Spit (0.31 m). The seiche nodal line runs from the base of the Dolzhanskaya Spit to Berdyansk approximately. A similar result is described in [9].

Observations from meteorological stations and hydro-posts have shown that water level oscillations with a period of approximately 24 hours occur most frequently. Such water level oscillations with a period of 23–25 hours are described in [5, 7]. Work [7] indicates that the dominant longitudinal free oscillations of the Sea of Azov have a period of approximately 24 hours. One of this mode's antinodes is located in Taganrog Bay, and the other is near Genichesk. This situation closely resembles the pattern presented in Fig. 2.

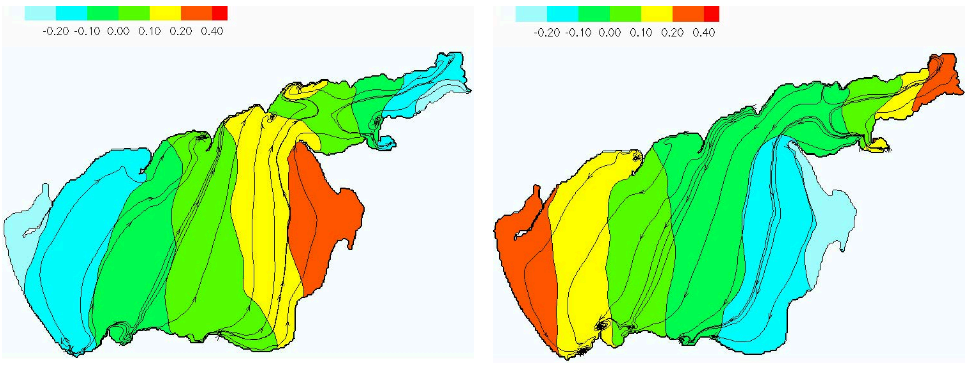


a *b*

F i g. 2. Resonant oscillations at changing wind (speed 2 m/s) direction from NE to SW at the end of: *a* – period *T* = 26.0 h, and *b* – semi-period *T*/2 = 13.0 h

With a wind oscillation period of 19.2 hours, the Sea of Azov exhibits a circulatory pattern of water motion with the formation of several circulation zones, similar to the pattern observed with a 26-hour period. In this case, however, the oscillation amplitude is smaller, equaling 0.25 m at maximum and −0.45 m at minimum.

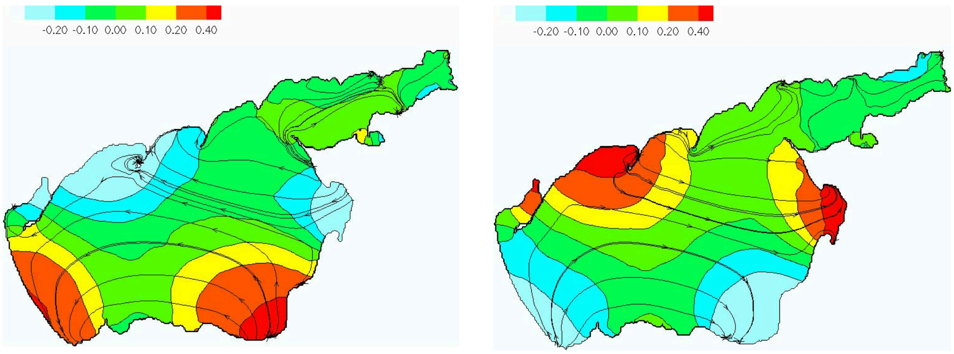
Wind oscillations with a period of 15.2 hours primarily cause translational water motion in the Sea of Azov (Fig. 3), revealing a two-node seiche. Two elevation maxima are observed in Taganrog Bay (0.39 m) and the Utlyuk Estuary (0.42 m), as well as a minimum in the Primorsko-Akhtarsk area (−0.44 m). These maxima and minima alternate with each other: a maximum in the Primorsko-Akhtarsk area and two minima in Taganrog Bay and the Utlyuk Estuary. One nodal line connects the Kerch Strait area with Berdyansk Bay. A second nodal line connects the Yeysk area with the Belosaraysk Spit.



a *b*

F i g. 3. Resonant oscillations at changing wind (speed 2 m/s) direction from NE to SW at the end of: *a* – period *T* = 15.2 h, and *b* – semi-period *T*/2 = 7.6 h

With a wind oscillation period of 6.4 hours (Fig. 4), the main water level oscillations occur шт the Sea of Azov. The maximum level is recorded in the Primorsko-Akhtarsk area and Obitochny Bay (0.85–1.0 m), while the minimum level is observed near the mouth of the Kuban River and the Arabat Spit (−0.55 m). After half the period (3.2 hours), the spatial distribution of level extremes reverses. One nodal line passes through the center of the Sea of Azov, stretching from the Primorsko-Akhtarsk area to Biryuchiy Island. The other two nodal lines are located at the boundary of Taganrog Bay and in its central part.



a *b*

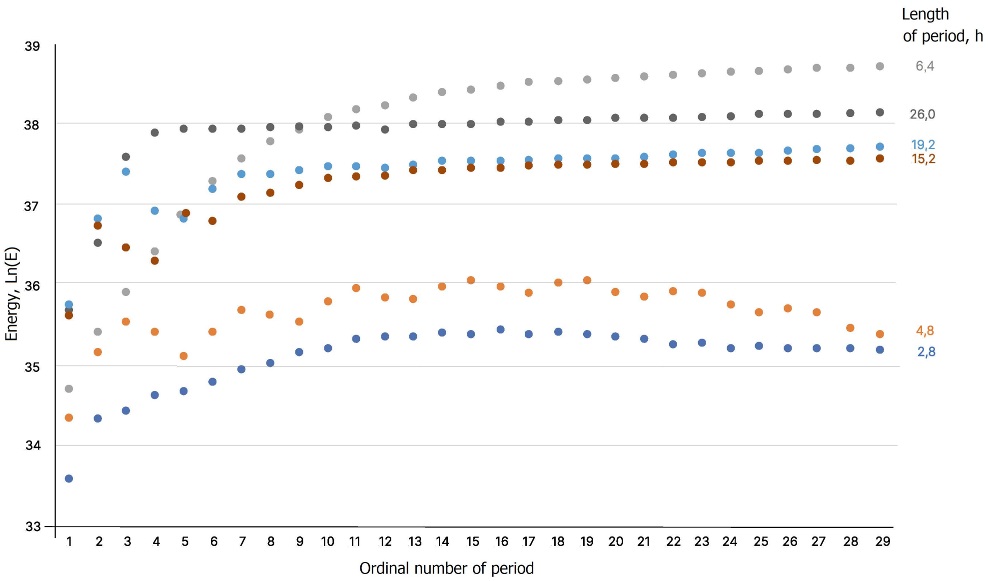
F i g. 4. Resonant oscillations at changing wind (speed 2 m/s) direction from NE to SW at the end of: *a* – period T = 6.4 h, and *b* – semi-period $T/2$ = 3.2 h

Further investigation of oscillations at peak frequencies revealed that, for wind oscillations with a 26-hour period, a sharp increase in total energy begins as early as the fourth period of forcing. The difference in total energy values for subsequent periods does not exceed 1%. For 15.2-hour oscillations, the sharp increase in energy begins in the sixth period. For 6.4-hour oscillations, the sharp increase in energy begins in the seventh period.

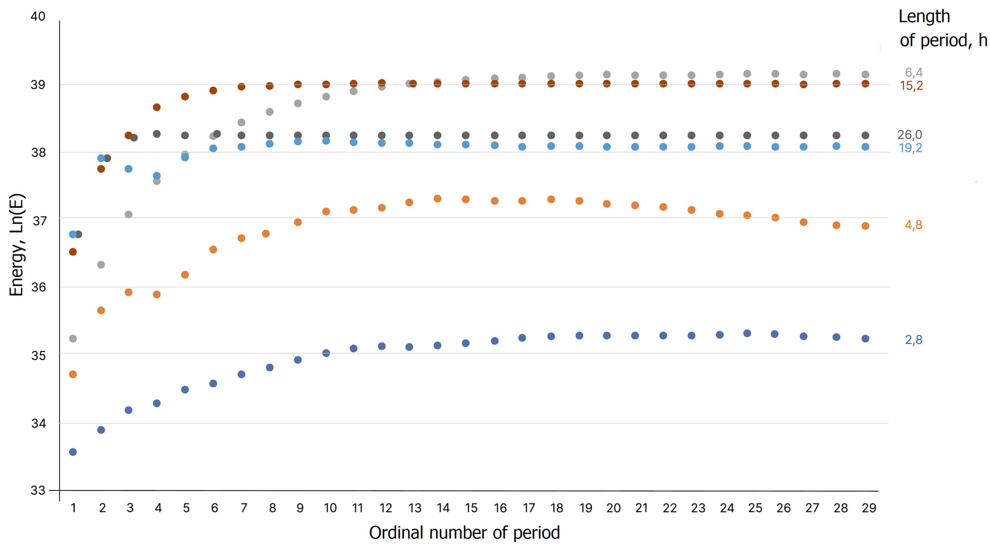
With continued external oscillatory forcing at periods of 6.4, 15.2, 26, and 19.2 hours, the total energy value steadily increases, indicating a possible onset of resonance (Fig. 5). Note that the 19.2-hour frequency was not previously considered resonant. For frequencies with oscillation periods of 2.8 and 4.8 hours, continued external forcing leads to a decrease in total energy, indicating oscillation damping.

When wind action alternates with calm, the patterns of water surface level distribution in the Sea of Azov resemble those observed in the previous scenario.

Forced water level oscillations in the Sea of Azov were also generated using atmospheric pressure oscillations. It was assumed that there was no wind forcing. Different pressure values related by a linear dependence were specified at the western (Arabat Spit) and eastern (Don River delta channels) boundaries of the sea. Atmospheric pressure was set to 750 mm Hg at one end of the Sea of Azov and 770 mm Hg at the other. As expected, the peak values of total energy correspond to pressure change frequencies that coincide with the frequencies of wind forcing oscillations (see Fig. 1): 6.4, 15.2, 19.2, and 26 hours. Notably absent are total energy spikes for periods of 9.2 and 11.6 hours, as well as anomalous energy behavior for periods shorter than 6.4 hours.



F i g. 5. Total energy for peak values of oscillation periods (h) under wind impact on the Azov Sea surface



F i g. 6. Total energy for peak values of oscillation periods (h) under the influence of atmospheric pressure on the Azov Sea surface

With continued atmospheric pressure acting on the sea surface at peak frequencies for 6.4, 15.2, 19.2, and 26-hour periods, the total energy value stabilizes, while for periods of 2.8 and 4.8 hours, the total energy decreases (Fig. 6). This confirms that the peak energy spikes for periods of 2.8 and 4.8 hours are not resonant.

Conclusions

The conducted numerical study demonstrated that significant water level oscillations in the Sea of Azov can be caused by relatively weak external forcing on the water surface. These oscillations are not necessarily resonant in nature, and significant disturbances to the water surface occur within the first four to seven periods of the external force's oscillation. Similar results were obtained for wind and atmospheric pressure forcing on the surface of the Sea of Azov.

The results of this study are consistent with those obtained by other authors, although they do not show exact correspondence. This discrepancy is due to the use of different mathematical models and differences in the approximation of the Sea of Azov itself, which affects the natural oscillation values of the model.

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About the author:

Alexey L. Chikin, Chief Researcher, Federal Research Center the Southern Scientific Center, Russian Academy of Sciences (41 Chekhov Ave., Rostov-on-Don, 344006, Russian Federation), DSc. (Phys.-Math.), **SPIN-code: 3845-9760**, **ORCID ID: 0000-0002-4065-010X**, **Scopus Author ID: 8240627300**, chikin1956@gmail.com

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