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Original article

Synchronization of Long-Wave Fluctuations of Sea Level in Adjacent Bays: The Example of the Kholmsk Bays (Sakhalin Island)

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Abstract
Purpose. The purpose of the study is to define the possibility of synchronizing long-wave fluctuations in the neighboring bays of the Kholmsk city based on the Van der Pol oscillator model, as well as the possibility for long waves to penetrate to the coastal waters adjacent to Kholmsk from the Moneron Island region and from the water areas near the cities of Gornozavodsk, Nevelsk, and Chekhov using *in-situ* observation data.
Methods and Results. The autonomous wave recorders ARW-14K installed in the Kholmsk bays, and the ARW-10 recorder located in the water areas nearby the cities of Gornozavodsk, Nevelsk, and Chekhov were used to obtain *in situ* data on sea level fluctuations. The measurement resolution of all the instruments is 1 s. The time series were researched through the spectral analysis of sea level fluctuations using the *Kyma* software. Calculation of the dispersion relation for the Stokes edge waves in the flat-sloping-bottom approximation has demonstrated the possibility of generating edge waves with the 8–9 min period in the water areas nearby the populated regions at the southwestern coast of Sakhalin Island. The reasons for unlimited growth of the phase difference of sea level fluctuations shown in the phase diagram for the Kholmsk-Severny and Trade Port bays were analyzed using a numerical solution of the Van der Pol equation.
Conclusions. It is established that a well pronounced wave process with the 8.27 min oscillation period is observed only in the water area nearby Gornozavodsk, and these waves do not reach the Kholmsk outer water area. The sea level fluctuations (period is approximately 8 min.) recorded by the tide gauge at Trade Port are the result of fluctuation interactions in the bays Trade Port and Kholmsk-Severny. A numerical solution of the problem on forced synchronization of a dynamic system subjected to a weak periodic stimulus using the Van der Pol equation has shown the possibility of unlimited growth of the phase differences between the phase of external force, i. e. the waves incoming from the neighboring Kholmsk-Severny Bay (period is 8.65 min), and the phase of natural oscillations in the Trade Port Bay (period is 4.7 min), which are obtained from *in situ* data. At a small value of the nonlinearity parameter, the oscillations of the Van der Pol oscillator are close to the harmonic ones, and in this case the phase difference of fluctuations changes abruptly and increases continuously.

Keywords: sea level fluctuations, edge waves, oscillation synchronization, Van der Pol oscillator

Acknowledgments: Numerical calculations, collection of *in situ* data, their processing, and subsequent analysis were carried within the framework of state assignment of the FSBSI IMGandG, FEB of RAS FWWM-2024-0002; analysis and interpretation of the results of *in situ* data processing, and numerical calculations were done within the framework of state assignment of the FSBSI FRC MHI FNNN-2024-0016.

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Introduction

It is well known that in the coastal zone of seas various types of long waves can be generated – shelf seiches and edge waves, and in bays, gulfs, and semi-enclosed water areas – seiches. Many researchers have studied these waves [1, 2] [1–6]. Each water area has its own features: waves of certain periods can be generated in it, and penetrating waves can be amplified in different ways. Therefore, it is necessary to investigate the properties of each specific water area associated with human economic activity.

The excitation of seiches – natural oscillations of water areas – in coastal harbors is often related to infragravity (IG) waves [7] and less frequently to atmospheric pressure fluctuations [8, 9], tsunamis [3] [10], earthquakes [11] and internal waves [12]. It should be noted that seiches with periods of several minutes, characteristic of IG waves, often dominate sea level fluctuations on oceanic coasts [7, 13, 14] and in harbors with small areas (less than one square kilometer) and depths of less than 12 m.

Waves arriving from the open ocean can be reflected from the shore as leaky waves and generate shelf seiches in the shelf zone, which acts as a kind of resonator [2]. These waves are observed quite often in our experimental studies [15, 16]; their periods range from several minutes to a few hours.

Near the marine shoreline, "trapping" of wave energy [2] is observed [17], which has a significant effect on coastal dynamics. In this case, the shelf zone plays the role of a waveguide, and trapped waves can propagate along it over large distances without considerable attenuation [2]. The principal type of barotropic wave trapping is gravitational trapping in the shallow-water area due to the decrease in the phase velocity of gravity waves with decreasing sea [1] depth, which results in the generation of edge waves.

The generation of the waves considered here is mainly facilitated by IG waves, which transfer their energy to other types of coastal waves through various mechanisms. IG waves themselves are produced by nonlinear interactions between wind waves, between wind waves and swell, as well as by reflection from the shore. Their periods range from 30 s to several minutes [2]. The dynamics of IG waves (e.g., nonlinear generation, dissipation, and trapping), which determine the energy levels of infragravity waves on beaches and in harbors, have not been fully studied and cannot yet be accurately predicted theoretically, while unsteady nonlinear Boussinesq models are still under development [7, 18].

According to the data of our investigations, sea level fluctuations with a period of ~ 8 min have been detected in the port bays of Kholmsk [19, 20], which can be attributed to seiche oscillations of these bays. The paper [21] notes that the source of long-wave fluctuations with the indicated period is long-wave resonators that accumulate and amplify the energy of trapped waves in the area of Moneron Island and on the shelf near the city of Chekhov. Numerical modeling of resonant oscillations in the harbor of the Kholmsk Trade Port did not reveal the presence of intense fluctuations in the indicated time interval [21].

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[1] Efimov, V.V., Kulikov, E.A, Rabinovich, A.B. and Fine, I.V., 1985. [*Ocean Boundary Waves*]. Leningrad: Gidrometeoizdat, 280 p. (in Russian).

[2] Rabinovich, A.B., 1993. *Long Gravitational Waves in the Ocean: Capture, Resonance, and Radiation*. Saint Petersburg: Gidrometeoizdat, 324 p. (in Russian).

[3] Lepelletier, T.G., 1980. *Tsunamis – Harbor Oscillations Induced by Nonlinear Transient Long Waves. Report No. KH-R-41*. California, Pasadena: California Institute of Technology.

Therefore, it was important for the authors of [20] to clarify whether the seiche oscillations in Trade Port Bay interact with those arriving from the Moneron Island region or from the city of Chekhov. In the present study, an assumption has also been made that the effect of oscillation synchronization is observed in Trade Port Bay with the fluctuations arriving at its entrance from the neighboring Kholmsk-Severny Bay, i.e., the interaction of oscillations in the bays takes place. It was decided to verify these assumptions; the results are presented in the present study.

To represent the wave motions considered here in the neighboring bays, we used the well-known Van der Pol equation, which describes the oscillations of a weakly nonlinear dynamic system and is one of the basic models in nonlinear dynamics.

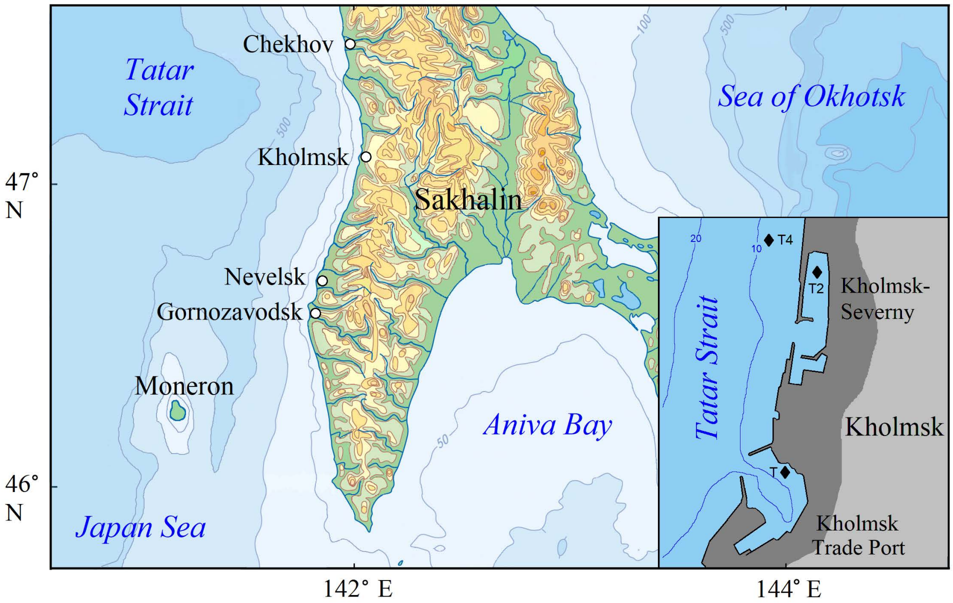
The present paper is aimed at studying the possibility for long waves to penetrate into the coastal water area adjacent to Kholmsk from the Moneron Island region and from the water areas nearby the cities of Gornozavodsk, Nevelsk, and Chekhov, as well as at studying the interaction between the Kholmsk bays and establishing the causes of the occurrence of sea level fluctuations with a period of ~ 8 min in them.

Materials and methods

Object of study. The water area of the coastal part of southern Sakhalin Island from Gornozavodsk to Chekhov, including the bays of the port of Kholmsk, is the object of study. A map of the water area is shown in Fig. 1.

The dimensions of the port bays are given in [20]; therefore, here we only indicate the distances between the wave observation points, which were obtained in 2008. The instruments were installed near Gornozavodsk at a depth of 2.5 m, and near Chekhov and Nevelsk – at a depth of ~ 15 m. The distance from Moneron Island to Gornozavodsk is 56.5 km, from Gornozavodsk to Nevelsk it is 12.5 km, from Gornozavodsk to Kholmsk it is 56.9 km, and from Kholmsk to Chekhov it is 42.8 km. The shelf width in the areas of the populated regions under consideration is approximately the same, ~ 40 km, with a bottom slope of approximately 0.0078. The shelf in the Kholmsk area is the narrowest and steepest near the western coast of Sakhalin; it widens both southward and northward.

Sea level observations. The study of sea level fluctuations was carried out in the vicinity of Chekhov and Nevelsk from June to September, and in Gornozavodsk in June–July 2008. Autonomous wave recorders of the ARW-10 model were used. Measurements in the Kholmsk area were performed from September 2022 to May 2023 using ARW-14K instruments (T, T2, T4 in Fig. 1) with serial numbers 152, 142, and 149. The accuracy of measuring bottom hydrostatic pressure by these instruments is identical; the difference lies in the power supply and the size of the housings. The measurement range of the level sensor is 0.5–100 m of water column, and that of the temperature sensor is from −4 to +40 °C. The measured hydrostatic pressure was subsequently recalculated into sea level fluctuations (waves) taking into account the attenuation of short waves with depth. The error is 0.06% of the upper measurement limit, and the pressure resolution is ±0.0003%. The measurement resolution of the instruments is 1 s.



F i g. 1. Map of the coastal water area of the southern Sakhalin Island. Sites of installation of T, T2 and T4 instruments in the port bays of Kholmsk are marked with black diamonds

<u>Methods for processing in situ observation data</u>. To analyze the observation data, the Kyma software was used. It was developed for visualization, comprehensive processing, and spectral analysis of large-volume sea level measurement data [22, 23]. The software performs the calculation of the spectral density of sea level fluctuations from a time series using the windowed Fourier transform.

When analyzing short-period waves with minute periods, the predicted tide was subtracted from the time series. The calculation algorithm based on the least squares method was developed at the Marine Physics Laboratory of the IMGG FEB RAS and has been repeatedly tested. The algorithm takes into account 35 astronomical tidal harmonics; their subtraction from the initial time series is performed using the LSMTM.exe application within the Kyma software.

Results and discussion

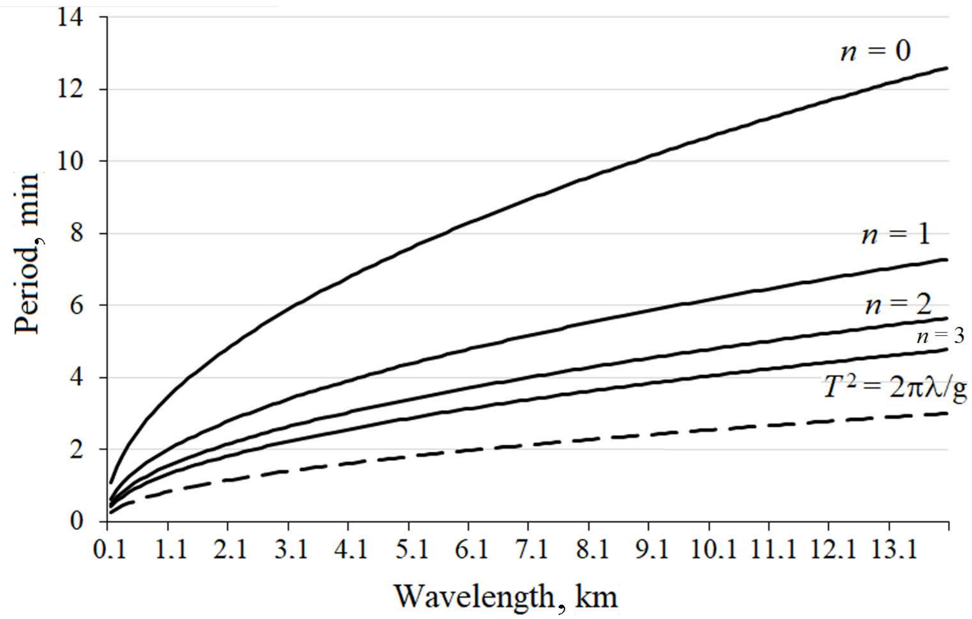
Since [21] assumes that the long-wave fluctuations with a period of ~ 8 min, recorded in Trade Port Bay, Kholmsk, are trapped waves coming from the regions of Moneron Island and the shelf near Chekhov, it was decided to test the possibility of generating such edge waves. In 2008, one wave recorder was placed near Chekhov. No instruments were installed near Moneron Island; therefore, the results of observations off the coast of Sakhalin Island – near Gornozavodsk and Nevelsk, where wave measurements were also carried out during that period – were used.

<u>Edge waves</u>. The analysis of the possibility of generating edge waves with periods of ~ 8 min was carried out taking into account that the bottom profile at a distance of up to 35 km from the shore is fairly flat, with an approximately

constant bottom slope, and the periods of trapped waves can be determined using the dispersion relation obtained in [24]:

$$\omega_n^2 = gk \sin[(2n + 1)\beta], \quad (1)$$

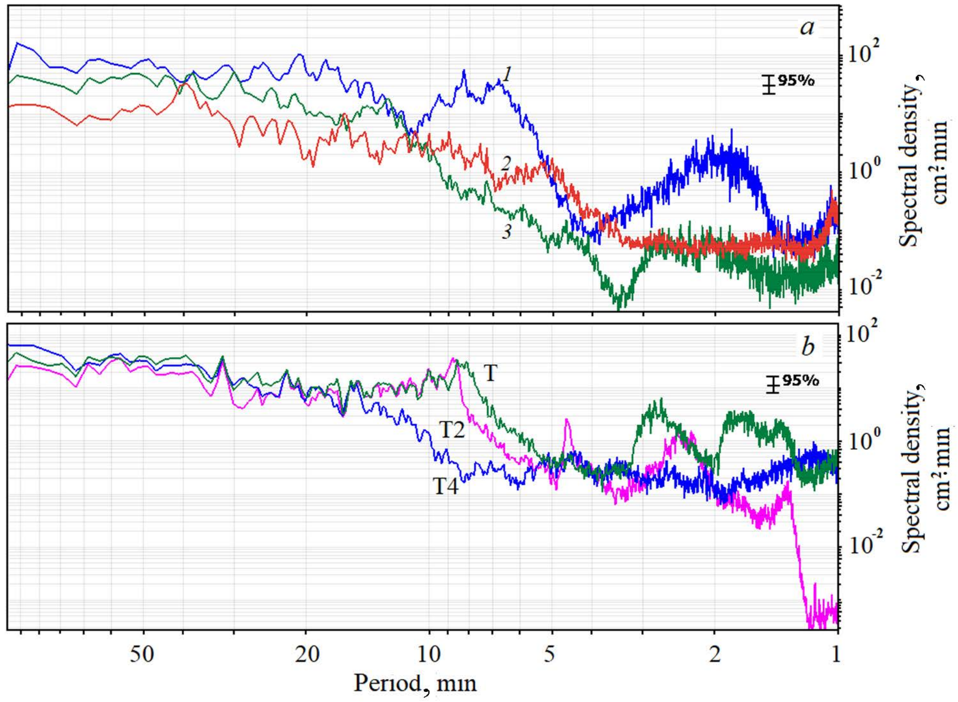
where ω_n is the frequency of the n^{th} edge wave mode; g is the acceleration due to gravity; k is the alongshore wave number; β is the bottom slope. The lower limit of the existence of edge waves is bounded by the value $k \geq \omega^2/g$ [25], and the number of modes of these waves for any bottom slope is always limited by $n \leq \pi/4\beta - 1/2$ [2]. The dispersion diagram calculated using expression (1) for the first four edge wave modes (Fig. 2) demonstrated the possibility of generating such waves with periods of ~ 8 min near Gornozavodsk, Nevelsk, and Chekhov. Consequently, if such edge waves are generated in real conditions, they can propagate to Kholmsk, penetrate into the bays, and transfer energy to seiches.



F i g. 2. Dispersion diagram of edge waves for the first four modes. Below the curve $T^2 = 2\pi\lambda/g$, there is a continuum of the Poincare waves

Such a possibility of penetration into bays of an edge wave propagating along the eastern coast of Shikotan Island was shown by the results of the study in [26]. This wave was recorded in Tserkovnaya and Dimitrova bays with a period close to the natural oscillation periods of these bays.

Let us verify whether the generation of edge waves with a period of ~ 8 min actually occurs in the water areas nearby the indicated populated regions. This can be done by calculating the spectral densities of sea level fluctuations using *in situ* observation data. The calculated spectral densities for the minute period range are shown in Fig. 3.



F i g. 3. Spectral densities of the measured sea level fluctuations for a minute range of periods: *a* – in the Gornozavodsk (*1*), Chekhov (*2*) and Nevelsk (*3*) regions in 2008; *b* – in the Kholmsk region in 2023 (measurements were carried out by instruments T, T2 and T4)

Analysis of the spectral density plots of sea level fluctuations shows that a well-pronounced wave process with an oscillation period of 8.27 min is observed only in the water area near Gornozavodsk (Fig. 3, *a*). In the water areas of other points, peaks in the period range of 7.5–9 min exceeding the confidence interval are absent. Significant peaks at these periods are also absent in the outer water area of Kholmsk (Fig. 3, *b*, Gornozavodsk area).

This refutes the conclusion of the paper [21], according to which the fluctuations with a period of ~ 8 min recorded in the records of instrument T (Fig. 1) in Trade Port Bay are trapped waves from the area near Chekhov. It is also doubtful that edge waves from Moneron Island can reach the water area of Gornozavodsk, since this island is located at a great distance from Sakhalin Island and does not share a common shelf with it. The waves generated in the water area near Gornozavodsk with a period of 8.27 min are edge waves, since the calculation of seiches for the open bay here between Cape Lopatin and Cape Bogdanovich showed the absence of natural oscillation modes in the period range of 7–9 min. However, these edge waves also do not reach the water area of Kholmsk, because otherwise they would be recorded in the intermediate water area of Nevelsk. Possibly, the wide Cape Lopatin prevents the propagation of these waves toward Kholmsk.

It can be concluded that in this situation, edge waves with a period of 8–9 min do not arrive in the Kholmsk water area, and oscillations with the indicated periods are not excited in it. Only one mechanism remains for the generation of oscillations with such a period in the Kholmsk bays – the interaction of oscillations in the Trade

Port Bay and Kholmsk-Severny Bay. This is also confirmed by the findings of [20], namely, the presence in the spectral density of sea level fluctuations in both bays of beats with a period of 294.5 min (4.91 h) arising from the interaction of modes with periods of 8.17 and 8.65 min. Moreover, a synchronous increase in oscillation amplitudes that are in phase is often observed in both bays (March 11, 2023).

Natural oscillations, Trade Port Bay. According to the website http://retromap.ru/1419537_z7_46.335550,142.22351&h=0, the dimensions of the Trade Port Bay, Kholmsk, are as follows: length to the ferry berth 732 m, length from the bay entrance to the wall 556 m, width 422 m, width of the narrow inner (apex) part 109 m, its length 350 m, width of the bay entrance 174 m, depth near the berth 2.9–9.5 m, average depth 6.5 m. Based on these data, we obtain estimates for the natural oscillation periods of this water area.

The theoretical periods of longitudinal natural modes for rectangular bays with an open entrance are calculated by formula [2]

$$T_n = \frac{4L}{(2n+1)\sqrt{gh}}, \quad (2)$$

where L is the bay length; h is its depth; g is the acceleration due to gravity; $n = 0, 1, 2, \ldots$.

As can be seen from formula (2), the value of the Helmholtz mode period (zeroth mode) can be estimated by the time T^* it takes a long wave to travel through the bay at speed $\sqrt{gh}$; it equals $4T^*$. Some researchers believe [27] that in practice the period is usually somewhat longer and attribute this to the wave reflection at the open boundary, which differs from the reflection from the shore.

The calculated values of the long-wave propagation time T^* and $4T^*$ for different parts of the Trade Port water area and different average depths are given in the table. Initially, the propagation time was calculated (at the average depth of the water areas) from the bay entrance to the opposite wall and for the inner (apex) part of the water area. In this case, an average depth value of 6.5 m was used. It is noteworthy that for the apex part, the time $4T^*$, corresponding to the period of the zeroth mode T_0, equals 3.76 min. According to the *in situ* data obtained with the instrument installed in the apex part of the bay, a period of ~ 3 min has been identified.

Time of long-wave propagation T^* and $4T^*$ for different parts of the Trade Port water area

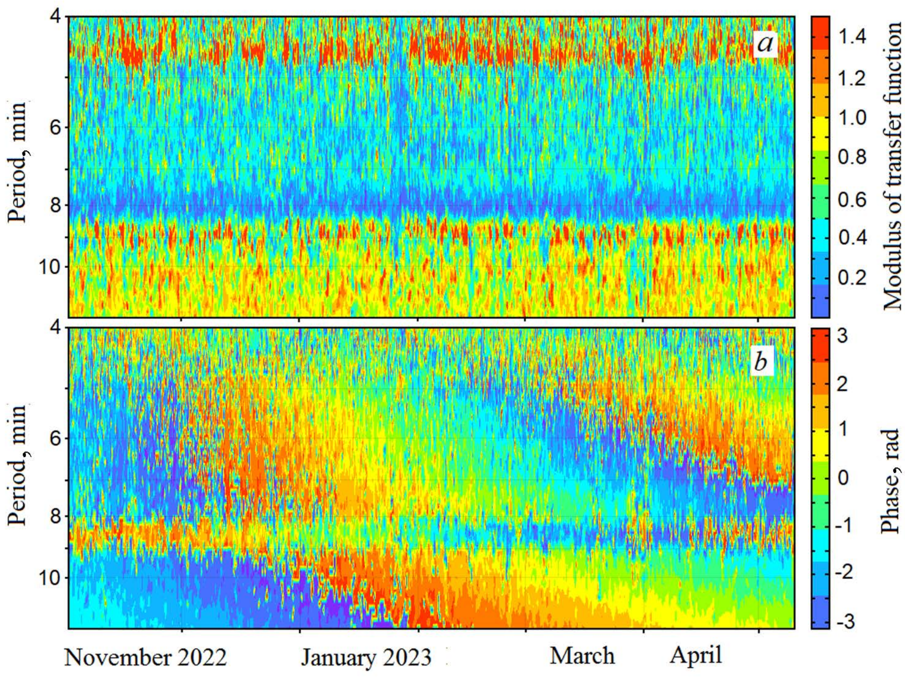
Water area	Distance L, m	Average depth according to the website, m	Average assumed depth, m	Propagation time T^* for a depth of 6.2 m, min	Propagation time T^* for a depth of 7.6 m, min	$4T^*$ for a depth of 6.2 m, min	$4T^*$ for a depth of 7.6 m, min
From the bay entranc to the wall	556	6.2	7.6	1.49	1.21	5.97	4.87
Apex part	350	6.2	7.6	0.94	0.76	3.76	3.07

Taking into account the oscillation period of 3 min, the average depth of the bay was corrected, amounting to ~ 7.6 m. The period of the calculated zeroth mode of the bay for this depth is 4.87 min, which is close to the theoretically calculated period of 4.7 min given in Table 2 of [20]. Thus, it can be confidently considered that the theoretical value of the zeroth mode period of the natural oscillations of the Trade Port Bay is ~ 4.9 min.

In [21], based on spectrogram estimates in the water areas of the southwestern coast of Sakhalin Island, it was assumed that sea level fluctuations with a period of ~ 8 min should appear in the records of the Kholmsk tide gauge, the source of which are long-wave resonators accumulating and amplifying the energy of trapped waves in the Moneron Island area and on the shelf near Chekhov. This suggests that fluctuations with the indicated period penetrate into the bay from outside. No references are made to the presence of natural oscillations in the Trade Port Bay with a period of ~ 8 min.

Analysis of the causes of the unlimited growth of the phase difference of oscillations

Let us consider the results of calculating the spectral characteristics from the data of instruments T and T2 in the port bays of Kholmsk. The diagrams of the transfer function and the phase difference of fluctuations between the stations (Fig. 4) show the presence of intense long-wave processes with periods of ~ 8.65 min. At the same time, fluctuations with the noted periods are also recorded from observation data in the Trade Port Bay itself, although according to calculations, the period of the natural oscillations of the Helmholtz mode is ~ 4.9 min.

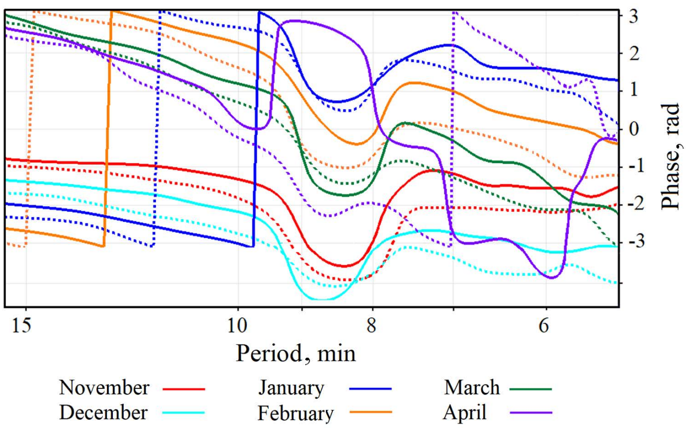


F i g. 4. Diagrams of transfer function (*a*) and phase difference (*b*) of sea level fluctuations based on the data of T and T2 instruments

On the transfer function diagram (Fig. 4, *a*), bands at periods of ~ 4.7 and ~ 8.65 min, which correspond to wave processes in the bays, are distinguished. On the coherence function diagram, these processes were weakly pronounced; therefore, the authors used the transfer function, which, as shown in [28], is intended for describing the relationship between two series when one series drives the other. In such cases, it is preferable to form the gain or transfer function from the cross spectrum and the squared coherence [4]. As can be seen in Fig. 4, *a*, the transfer function proved informative for the case under consideration.

The transfer function, called the operator transfer function, is a complex function of frequency $k(j\omega) = K(\omega)e^{-j\omega}$. Its magnitude $K(\omega)$ and argument $\varphi(\omega)$ show, respectively, how the amplitude and phase of each harmonic component of the spectrum of the transformed function change after the action of a linear operator on it [29]. In harmonic analysis, the magnitude of the transfer function $K(\omega)$ is called the amplitude-frequency characteristic, and the argument $\varphi(\omega)$ is called the phase-frequency characteristic of the operator.

On the diagram of the phase difference of fluctuations between Kholmsk-Severny Bay and Trade Port Bay (Fig. 4, *b*), a band at the period of ~ 8.65 min is clearly distinguished over the entire observation interval. It is seen that the phase change of this wave process occurs continuously. It should be noted that with two-day averaging, the phase difference growth is fairly uniform. With a more detailed calculation over two-day segments every 15 days (December 2022 – April 2023), a constant increase in the phase difference is observed (Fig. 5), except for the plots for mid-March and early April. An assumption was made that the constant phase difference growth is apparently related to the circumstance that the oscillator (the resonant water area of Trade Port Bay) with a natural oscillation period of 4.7 min is synchronized by waves with a period of 8.65 min arriving from Kholmsk-Severny Bay. No other wave processes with periods in the range of 7.5–9 min have been detected near the port bays, as shown above.



F i g. 5. Difference of fluctuation phases calculated from the two-day time series with the 15-day interval based on the data from T and T2 instruments. Solid lines show the month beginning, and dashed ones – its middle

[4] Denman, K.L., 1975. *Spectral Analysis: A Summary of the Theory and Techniques*. Technical Report No. 539. Environment Canada, Fisheries and Marine Service, 37 p.

Model

According to [30–34], the analysis of the current state of research on dynamical systems and phase synchronization has shown that to explain the noted unlimited growth of the phase difference, it is necessary to consider the synchronization of a regular self-oscillatory system (the bay water area) by an external periodic force – the incoming wave. An oscillator is considered to be synchronized by an external periodic stimulus if its observed frequency becomes equal to the frequency of the external signal [5].

To explain the discovered continuous growth of the phase difference, the possibility of describing such a phenomenon using the Duffing, Mathieu, and Van der Pol equations was considered [35, 36]. The most appropriate model for the case under consideration is the case of a weak and arbitrary periodic stimulus acting on a regular self-oscillatory system – the Van der Pol oscillator. Verification of other description options confirmed the correctness of the choice made.

If the amplitude of the external force is small ($\varepsilon << 1$), then the problem of forced synchronization of the system can be solved within the framework of the following model [5]:

$$\dot{\varphi} = \omega_0 + \varepsilon q(\varphi - \omega t), \tag{3}$$

where φ is the oscillation phase of the system; t is the time; $\omega_0 = 2\pi/T_0$ is the frequency of periodic oscillations of the oscillator (T_0 is the limit cycle of the autonomous system); q is a periodic function of φ; $\omega = 2\pi/T$ is the frequency of the external force with amplitude ε.

If we introduce a new variable θ (the difference between the phase of the external force ωt and the fluctuation phase φ) and average equation (3) over the period of the external force, we obtain, according to [5]

$$\dot{\theta} = \delta - \varepsilon q(\theta), \tag{4}$$

where the frequency detuning $\delta = \omega_0 - \omega$. In the simplest case of quasi-harmonic oscillations, $q(\theta) = \sin\theta$, and equation (4) takes the form

$$\dot{\theta} + \varepsilon \sin\theta = \delta. \tag{5}$$

If we introduce the parameter $\Delta = \delta/\varepsilon$ and a new time $t' = \varepsilon t$, equation (5) can be written as

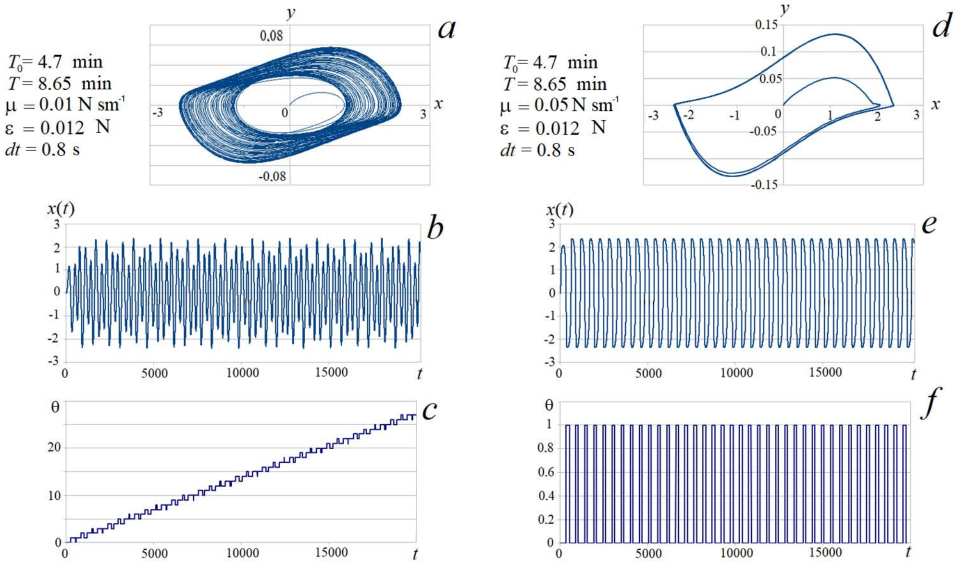
$$\dot{\theta} + \sin\theta = \Delta. \tag{6}$$

Analysis of equation (6) showed that at $\Delta = 1$, a bifurcation occurs [37]; the equilibrium states merge, and there is one equilibrium state with the coordinate $\bar{\theta} = \pi/2$. In the region $|\Delta| > 1$, there are no equilibrium states, and the phase difference increases without limit. The character of the phase difference growth depends on the parameter $\gamma = |\Delta| - 1$. If $|\Delta|$ is slightly greater than 1, then the growth $\theta(t)$ represents an alternation of relatively long intervals of an almost unchanged phase difference with preservation of the synchronous regime for some time, and short intervals of its rapid growth. In this case, the phase

[5] Osipov, G.V. and Polovinkin, A.V., 2005. [*Synchronization with External Periodic Impact*]. Nizhniy Novgorod: NNGU, 78 p. (in Russian).

growth is significantly nonuniform over different time intervals. As γ increases, the length of intervals of almost constant phase becomes smaller, and at a sufficiently large value of γ, the phase increases substantially uniformly.

The obtained numerical solution of equation (6) is shown in Fig. 6, *c*. It demonstrates the unlimited growth of the phase difference between the phase of the external force (i.e., the waves incoming from the neighboring Kholmsk-Severny Bay with a period of 8.65 min) and the phase of natural oscillations of the water in Trade Port Bay with a period of 4.9 min. The growth of the phase difference occurs linearly, and small jumps are associated with the discreteness of the calculation.



F i g. 6. Phase portraits (*a*, *d*), time series of oscillator vibrations (*b*, *e*) and phase difference (*c*, *f*) for the system described by equation (6) for the coefficient of viscous friction μ = 0.01 N s/m (*a*, *b*, *c*) and μ = 0.05 N s/m (*d*, *e*, *f*)

Also, for the parameters obtained from *in situ* experiments, a numerical calculation of the Van der Pol oscillator was performed, described by the system of ordinary differential equations:

$$\dot{x} = y,$$
$$\dot{y} = -\omega_0^2 x + \mu[(1 - x^2)y + 2\varepsilon \sin \omega t], \quad (7)$$

where ω_0 is the natural frequency of the oscillator; $\mu \geq 0$ is the nonlinearity parameter (viscous friction of the system), governing the shape of oscillations; ε and ω are the amplitude and frequency of the external force, respectively.

For the calculations, the values of the oscillation periods obtained from *in situ* observations were used: $\omega 0$ = 0.0223 rad/s ($T0$ = 4.7 min); ω = 0.0121 rad/s (T = 8.65 min); μ = 0.01 N s/m and μ = 0.05 N s/m; ε = 0.012 N (0.5 of the external force amplitude); the integration step is 0.8 s. Fig. 6 presents the results of calculations for two values of the nonlinearity parameter μ. The legend shows the following parameters: T_0 is the oscillation period of the water area; T is the period of incoming waves; ε is the amplitude of the external force; dt is the integration step. On the phase difference axes, the scale division is $2.5 \cdot 10^{-4}$ rad.

In both cases, there is a single attractor in the phase plane, with limit cycles varying in shape. At a smaller value of the nonlinearity parameter (μ = 0.01 N s/m), the oscillations of the Van der Pol oscillator are fairly close to harmonic ones (Fig. 6, *a*), and in this case the phase difference of fluctuations grows continuously (Fig. 6, *c*). For a fivefold greater value of the nonlinearity parameter (μ = 0.05 N s/m), the shape of the phase portrait (Fig. 6, *d*) corresponds to a strongly nonlinear oscillator, and with a further increase in μ, the oscillation shape of the oscillator changes from the harmonic type to the relaxation type. The phase difference of fluctuations in this case does not grow (Fig. 6, *f*).

A significant difference should be noted in the rate of approaching the limit cycle for weak and strong nonlinearity. In the case of strong nonlinearity, a rapid convergence of oscillations to the limit cycle is observed (Fig. 6, *d*), whereas in the case of weak nonlinearity, the convergence is slow, and the thickness of the attractor line is larger (Fig. 6, *a*). This property determines the differences in the time of synchronization onset in these systems.

Moreover, since both the phase diagram calculated from *in situ* data (Fig. 4, *b*) and the plots of the phase difference of fluctuations (Fig. 5) show a constant phase growth, it can be concluded that for the actually observed synchronization in Trade Port Bay, the oscillations of the dynamic system are close to harmonic, weakly nonlinear ones with unlimited growth of the phase difference.

Conclusion

The previously made assumption that long-wave fluctuations with a period of about 8 min recorded in the tide gauge records in Trade Port Bay, Kholmsk, could be trapped waves coming from the Moneron Island region and the shelf near Chekhov was verified.

The calculation of the dispersion relation for the Stokes edge waves in the flat-sloping-bottom approximation has demonstrated the possibility of generating edge waves with an 8–9 min period in the water areas nearby the populated regions of Gornozavodsk, Nevelsk, and Chekhov. At the same time, the analysis of the spectral densities of sea level fluctuations near these points made it possible to detect a well-pronounced wave process with an 8.27 min oscillation period only in the water area near Gornozavodsk. Significant spectral peaks at periods of 8–9 min are also absent in the outer water area of Kholmsk. Thus, it was established that the fluctuations with a period of about 8 min recorded in the tide gauge records in Trade Port Bay are the result of the interaction of fluctuations in Trade Port Bay and Kholmsk-Severny Bay.

The analysis of the causes for the unlimited growth of the phase difference of sea level fluctuations observed in the phase diagram for the Trade Port and Kholmsk-Severny bays was carried out. An assumption was made that the phase growth is most probably related to the fact that the oscillator – the resonant water area of the Kholmsk Trade Port, having a natural oscillation period of 3.0 min – is synchronized by waves arriving from the Kholmsk-Severny Port Bay with a period of 8.65 min, since no other wave processes with close periods have been detected near the port bays.

The problem of forced synchronization of a dynamic system subjected to a weak periodic stimulus was considered. The numerical solution of the differential phase equation has shown the unlimited growth of the phase difference between the phase of the external force – the waves incoming from the neighboring Kholmsk-Severny Bay with a period of 8.65 min – and the phase of natural oscillations of Trade Port Bay with a period of 4.7 min. In this case, the growth of the phase difference occurs linearly with preservation of the synchronous regime.

A numerical calculation of the Van der Pol oscillator was performed using the data obtained in *in situ* experiments for two values of the nonlinearity parameter: $\mu = 0.01\ N$ s/m and $\mu = 0.05\ N$ s/m. In both cases, the phase portraits represent a single attractor with limit cycles differing in shape. At a smaller value of the nonlinearity parameter, the oscillations of the Van der Pol oscillator are close to harmonic ones, and the phase difference of fluctuations in this case grows continuously. For a greater value of the nonlinearity parameter, the shape of the phase portrait corresponds to a nonlinear oscillator. This allows us to conclude that for the actually observed synchronization, the oscillations of the dynamic system are close to harmonic, weakly nonlinear ones.

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